

Reports of the Department of Geodetic Science

Report No.245

(NASA-CR-156970) DESIGN OF EXPERIMENT FOR EARTH ROTATION AND BASELINE PARAMETER DETERMINATION FROM VERY LONG BASELINE INTERFEROMETRY (Ohio State Univ. Research Foundation) 123 p HC A06/MF A01 CSCL 08F G3/43 16679 N78-22440 Unclas

**DESIGN OF EXPERIMENT FOR EARTH ROTATION
AND BASELINE PARAMETER DETERMINATION
FROM VERY LONG BASELINE INTERFEROMETRY**



by

Athanasios Dermanis

Prepared for the
National Aeronautics and Space Administration
Washington, D.C.

Grant No. NGR 36-008-204
OSURF Project No.783820



The Ohio State University
Research Foundation
Columbus, Ohio 43212

February, 1977

Reports of the Department of Geodetic Science

Report No. 245

DESIGN OF EXPERIMENT FOR EARTH ROTATION AND
BASELINE PARAMETER DETERMINATION FROM
VERY LONG BASELINE INTERFEROMETRY

by

Athanasios Dermanis

Prepared for the
National Aeronautics and Space Administration
Washington, D.C.

Grant No. NGR 36-008-204
OSURF Project No 3820-A1

The Ohio State University
Research Foundation
Columbus, Ohio 43212

February, 1977

PRE FACE

OSURF Project 3820-A1 is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University, and under the technical direction of Mr. James P. Murphy, Special Programs, Office of Applications, Code ES, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

The VLBI data and other pertinent information received from Mr. Peter F. Mac Doran of the Jet Propulsion Laboratory in connection with this investigation is gratefully acknowledged.

TABLE OF CONTENTS

	Page
Preface	ii
1. Introduction	1
2. VLBI—Basic Technique and Mathematical Model	3
3. Earth Rotation Parametrization	6
4. Linearization of Observation Equations	11
5. Coordinate System Definition and Inner Constraints	16
6. Remarks Related to Design of Experiment Concepts	19
7. Simulation and Adjustment Philosophy	25
8. Arrangement of Experiments	28
9. Results and Conclusions	35
References	67
Appendix A: Inner Constraints and Their Relation to the "Geometry" of the Pseudoinverse of an Operator	68
Appendix B: Algorithm for First-Order Partitioned Linear Regression Including Inner Constraints	72
Appendix C: Computer Programs	76

LIST OF FIGURES

	Page
2.1 The geometry of VLBI observations	4
6.1 Geometric loci of parameter sensitivity vectors	23
9.1 Geographic locations of stations and baselines in Experiments 1 through 10	34
9.2 Recovery of earth rotation parameters, Experiments 1 - 10	39
9.3 Recovery of baseline lengths and angles, Experiments 1 - 10	50
9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2)	61
9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2)	62
9.6 Variation of earth rotation parameter recovery with earth step	63
9.7 Variation of baseline length and angle recovery with earth step	65
A.1 The geometry of the pseudoinverse operator	70

LIST OF TABLES

6.1 Sensitivity of Baseline - Radio Source Extreme Configurations with Respect to Various Parameters	21
8.1 Present and Prospective VLBI Station Locations and Their Approximate Coordinates	29
8.2 Fictitious Stations Used in Experiments 6 and 9	30
8.3 Experiments and Participating Baselines	31

1. Introduction

Very Long Baseline Interferometry (VLBI) is one of the new techniques which will probably dominate geodesy and geophysics in the near future. Its main advantage lies in the fact that it brings the accuracy of direction measurements to a level previously possible only for range measurements. This closes the gap between powerful range determination techniques such as laser ranging and the much less accurate determination of directions through photographic tracking of artificial earth satellites.

The technique is geometric in the sense that the observations are independent of the gravity field of the earth. However, the "orbits" of the observed extragalactic radio sources with respect to an earth-fixed system are dominated and perturbed by the rotation of the earth with respect to inertial frame. This allows the determination of polar motion, precession-nutation and length-of-the-day variations, and the technique becomes also "dynamic" in this respect.

The capability of determining the geometry of a network of stations within a short time interval and with a centimeter level accuracy also allows the study of the variation of network geometry with time caused by earth tides and other periodic or secular station drifts.

The obvious importance of VLBI for both geodetic and geophysical applications is only limited by the effect of atmospheric refraction and unwanted noise associated with instrumentation [Shapiro and Knight, 1970, Section 3]. At present the development of the technique is at the stage of establishing its capabilities by direct comparison with classical surveys in relatively short baselines where geophysical effects are negligible, and of making the system more transportable.

In the present work the effect of nonwhite instrumentation noise, atmospheric refraction and earth tides are ignored, and the emphasis is on the possibility of recovering earth rotation and network geometry (baseline) parameters. It is assumed that nonwhite instrumentation noise is either eliminated or separately determined through calibration except for a linear effect which cannot be

distinguished from clock errors included in the adjustment parameters. Errors in the determination of atmospheric refraction are assumed to be either absent or uncorrelated in which case their effect is incorporated in the variance of the considered instrumentation white noise. The effect of earth tides on the variation of station coordinates is assumed to be calculated from separate information and subtracted from the actual observations.

The numerical simulated experiments performed here are set up in an environment where station coordinates vary with respect to inertial space according to a simulated earth rotation model similar to the actual but unknown rotation of the earth.

In Chapter 2 the basic technique of VLBI and its mathematical model are presented. The parametrization of earth rotation chosen is described in Chapter 3, and the resulting model is linearized in Chapter 4. Cartesian station coordinates are not estimable quantities in the analysis of VLBI observations. It is possible to use a model where only estimable quantities are considered (see, for example, [Arnold, 1974]). However, the choice of Cartesian coordinates leads to a simpler model, and the problem of coordinate system definition is resolved in Chapter 5 with the use of inner constraints. A simple analysis of the geometry of the observations in Chapter 6 leads to some useful hints on achieving maximum sensitivity of the observations with respect to the parameters considered. The basic philosophy for the simulation of data and their analysis through standard least squares adjustment techniques is presented in Chapter 7.

The main objective of the present work is the exploration of the capabilities of VLBI for the recovery of earth rotation and baseline parameters. For this purpose, a number of characteristic network designs based on present and candidate station locations is chosen in Chapter 8. The results of the simulations for each design are presented in Chapter 9 together with a summary of the conclusions.

2. VLBI—Basic Technique and Mathematical Model

Very Long Baseline Interferometry (VLBI) is a technique for observing time intervals and/or their derivatives at two widely separated antennas between the two instants of reception of the same wavefront emitted from extragalactic radio sources. The fundamental difference with respect to classical radio (short baseline) interferometry is in the fact that the antennas are not directly connected by cables for the comparison of the received signals. Sufficiently stable atomic frequency standards at each site make it possible to record the signals as functions of time. The recorded signals are then crosscorrelated afterwards in a computer. The fact that maximum correlation occurs when the two recordings are shifted in time by an amount equal to the wave travel time between the two instances of reception, makes it possible in principle to determine this time interval as well as its time derivative. Extragalactic radio sources are sufficiently far away for the wavefront to be considered as a plane and thus from the known velocity of the wave (velocity of light) the geometric distance corresponding to the observed time delay can be calculated.

The crosscorrelation procedure is described in detail in a series of articles by Thomas (1972a, 1972b and 1973), and we shall be concerned here only with the geometry of the observations. In Fig. 1, XYZ is a geocentric Cartesian reference frame fixed with respect to the radio sources (assumed to be an inertial frame). A certain wavefront arrives at station i at epoch t_1 , when the position vector of the station is $X_i(t_1)$, and at the station j at a slightly different epoch t_2 when the position vector of this station is $X_j(t_2)$. The true time delay $t_2 - t_1$ is the travel time corresponding to the projection D_{ij} of the retarded baseline $X_j(t_2) - X_i(t_1)$ on the direction of the radio source. If e is the unit vector in the radio source direction and c the velocity of light,

$$D_{ij} = c (t_2 - t_1) = [X_j(t_2) - X_i(t_1)] \cdot e \quad (2-1)$$

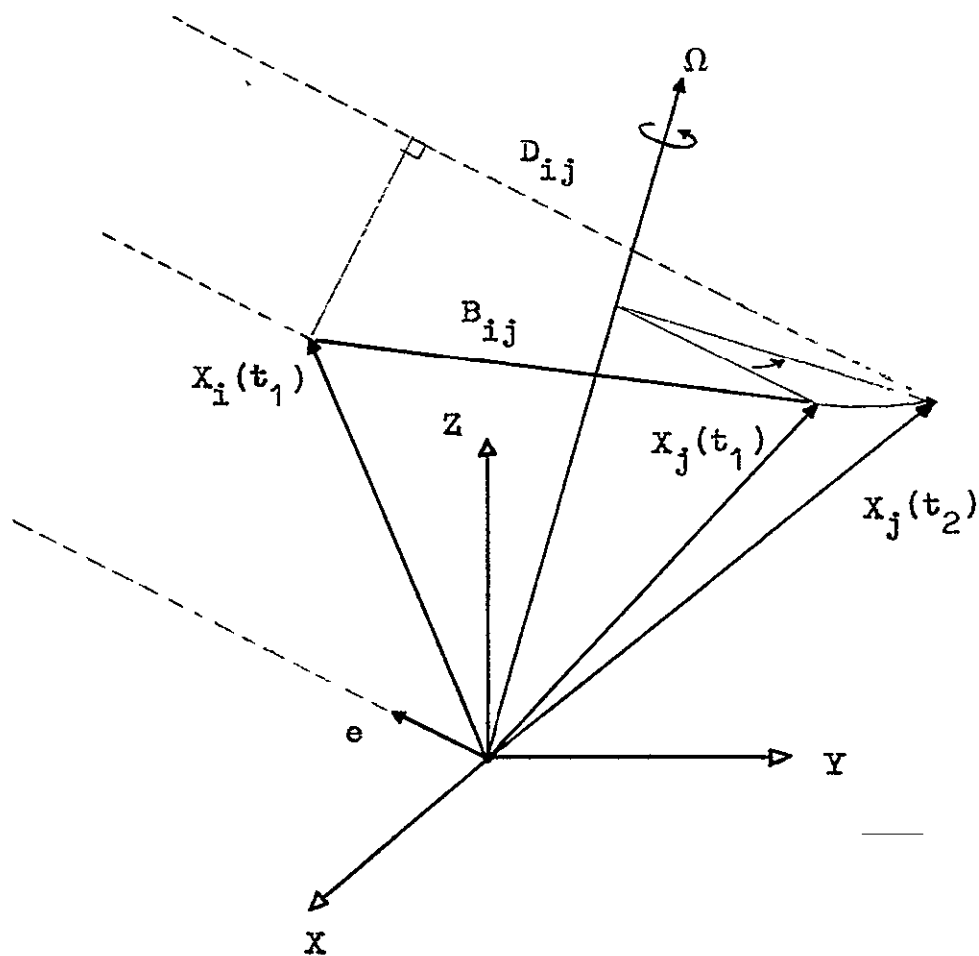


Fig. 2. 1 The geometry of VLBI observations .

Since $(t_2 - t_1)$ is a very short time interval, the time variation of the position vector $X_j(t)$ may be assumed to be solely due to the earth's rotation. Therefore

$$X_j(t_2) = X_j(t_1) + \int_{t_1}^{t_2} \Omega(t) \times X_j(t) dt \quad (2-2)$$

where $\Omega(t)$ is the rotational velocity vector of the earth. $\Omega(t)$ is practically constant over the short time interval $(t_1 - t_2)$, and thus, by setting $\Omega(t) = \Omega(t_1)$

$$X_j(t_2) = X_j(t_1) + (t_2 - t_1) [\Omega(t_1) \times X_j(t_1)] \quad (2-3)$$

Setting $B_{ij}(t) = X_j(t) - X_i(t)$ and $V_j(t) = \Omega(t) \times X_j(t)$, one obtains

$$D_{ij} = B_{ij}(t_1) \cdot e - \frac{1}{c} D_{ij} V_j(t_1) \cdot e \quad (2-4)$$

If approximate values of $\Omega(t_1)$, $X_j(t_1)$ and e are known with sufficient accuracy such that the corresponding uncertainty in the retardation effect

$$\delta D_{ij} = \frac{1}{c} D_{ij} V_j \cdot e \quad (2-5)$$

is negligible, one obtains, by setting $d_{ij} = D_{ij} + \delta D_{ij}$ and switching to matrix (column vector) notation, for an observation from the ij baseline to the radio source p at epoch t_k , the much simpler model

$$d_{ijpk} = B_{ij}^T(t_k) e_p \quad (2-6)$$

Assume that d_{ijpk} has been corrected for effects giving rise to discrepancies between modeled and true distance, e.g., atmospheric errors, relativistic effects, aberration, except for clock errors. Assuming that the two local oscillators at stations i and j differ only by a constant offset C_{oij} (at some epoch t_o) and a linear drift C_{ij} , the model becomes

$$d_{ijpk} = B_{ij}^T(t_k) e_p + c[C_{oij} + C_{ij}(t_k - t_o)] \quad (2-7)$$

Since the inner product of two vectors is invariant with respect to the choice of reference frame, the model may be rewritten

$$d_{ijpk} = b_{ij}^T e_p^*(t_k) + c[C_{oij} + C_{ij}(t_k - t_o)] \quad (2-8)$$

with

$$\mathbf{b}_{ij}^T = [x_j - x_i, y_j - y_i, z_j - z_i] \quad (2-9)$$

where \mathbf{b}_{ij} and \mathbf{e}_p^* are the baseline and the radio source direction unit vector, respectively, with respect to an earth-fixed reference frame x, y, z . One also has

$$\mathbf{e}_p^*(t_k) = \mathbf{M}(t_k) \mathbf{e}_p \quad (2-10)$$

where $\mathbf{M}(t)$ is the earth rotation matrix of transformation from the inertial to the earth-fixed frame. The model with respect to the earth-fixed reference frame becomes

$$\mathbf{d}_{ijpk} = \mathbf{b}_{ij}^T \mathbf{M}(t_k) \mathbf{e}_p + c[C_{\alpha ij} + C_{ij}(t_k - t_0)] \quad (2-11)$$

Further development of the model depends on the particular parametrization chosen for the rotation of the earth.

3. Earth Rotation Parametrization

The most straightforward way of parametrizing earth rotation is that of expressing the inertial to earth-fixed system transformation matrix $\mathbf{M}(t)$ in terms of three rotation angles at every epoch t . The most logical choice seems to be that of the Eulerian angles φ, θ, ψ , defined as

$$\mathbf{M}(t) = \mathbf{R}_3[\varphi(t)] \mathbf{R}_1[\theta(t)] \mathbf{R}_3[\psi(t)] \quad (3-1)$$

where $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3$ denote conventional rotation matrices about the x, y, z axes, respectively. The rotating earth can be viewed as a dynamical system whose state evolves in time according to a second-order differential equation of the form

$$\frac{d^2 \mathbf{E}}{dt^2} = \mathbf{f}(\mathbf{E}, t) \quad (3-2)$$

where $\mathbf{E} = [\varphi, \theta, \psi]^T$. If such equations of motion could be exactly known and solved, their solution

$$\mathbf{E}(t) = \mathbf{F}(\mathbf{E}(t_0), \dot{\mathbf{E}}(t_0), t) \quad (\dot{\mathbf{E}} = d\mathbf{E}/dt) \quad (3-3)$$

would provide a parametrization of the transformation matrix

$$M(t) = M(\varphi_0, \theta_0, \psi_0, \dot{\varphi}_0, \dot{\theta}_0, \dot{\psi}_0, t_0, t) \quad (3-4)$$

in terms of the six initial values (integration constants):

$$\begin{aligned} \varphi_0 &= \varphi(t_0) & \theta_0 &= \theta(t_0) & \psi_0 &= \psi(t_0) \\ \dot{\varphi}_0 &= \left. \frac{d\varphi}{dt} \right|_{t_0} & \dot{\theta}_0 &= \left. \frac{d\theta}{dt} \right|_{t_0} & \dot{\psi}_0 &= \left. \frac{d\psi}{dt} \right|_{t_0} \end{aligned}$$

However, such an approach is not practical because of uncertainties surrounding the current knowledge of the earth's rotation. The alternative is the representation of the functions $\varphi(t)$, $\theta(t)$, $\psi(t)$ in terms of a finite number of parameters to be estimated from the observations. Such representations as polynomials, trigonometric series, etc. can sufficiently approximate a wide class of functions, provided that appropriate number of terms is included. The efficiency of the approximation can be judged only when the function to be approximated is known over some time interval, while in our case an unknown function is to be approximated. The unknown approximation error (representation error, modeling error) may well be significant compared to the observational errors, thus affecting the validity of the final parameter error estimates based upon real data analysis.

Among various finite parameter representations, the simplest perhaps is that of representing a continuous bounded function over an interval by a step function, which has a constant value over each subinterval, into which the original time interval has been partitioned.

To formally represent a step function, we introduce the characteristic or indicator function χ_A of a set A , defined as follows:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \quad (3-5)$$

Let $[a, b]$ be the original interval and

$$a = t_0 < t_1 < t_2 < \dots < t_{n-1} < t_n = b$$

If χ_i denotes the characteristic function of the set $[t_{i-1}, t_i]$ ($i = 1, 2, \dots, n-1$) and χ_n that of $[t_{n-1}, t_n]$, a step function with respect to the above partition of $[a, b]$ can be written in the form

$$y(t) = \sum_{i=1}^n \alpha_i \chi_i(t) \quad (3-6)$$

A continuous bounded function $f(t)$ can be approximated arbitrarily well by a step function $y(t)$ with an appropriate choice and increase in the number of subintervals. In the case of this investigation, the functions to be approximated are the Eulerian angles $\varphi(t)$, $\theta(t)$, $\psi(t)$, and the effectiveness of step function approximation may be increased if the known parts of φ , θ , ψ are filtered out. Let $\varphi^0(t)$, $\theta^0(t)$, $\psi^0(t)$ be the "reference" Eulerian angles as computable from classical astronomy. Then the earth rotation transformation matrix may be represented as

$$M(t) = R_3[\varphi^0 + \delta\varphi] R_1[\theta^0 + \delta\theta] R_3[\psi^0 + \delta\psi] \quad (3-7)$$

where $\delta\varphi(t)$, $\delta\theta(t)$ and $\delta\psi(t)$ are step functions. An alternative parametrization may be of the form

$$M(t) = M^0(t) \delta M(t) = R_3(\varphi^0) R_1(\theta^0) R_3(\psi^0) R_3(\delta\varphi) R_1(\delta\theta) R_3(\delta\psi) \quad (3-8)$$

Traditionally, earth rotation representations are separated into three parts with the help of the instantaneous rotation vector $\omega(t)$ related to the Eulerian angles through Euler's geometric equations

$$\begin{aligned} \omega_x &= \sin \theta \sin \varphi \dot{\psi} + \cos \varphi \dot{\theta} \\ \omega_y &= \sin \theta \cos \varphi \dot{\psi} + \sin \varphi \dot{\theta} \\ \omega_z &= \cos \theta \dot{\psi} + \dot{\varphi} \end{aligned} \quad (3-9)$$

where ω_x , ω_y , ω_z are the rotation vector components with respect to the earth-fixed system. The variation in the direction of the vector ω with respect to the inertial system constitutes precession and nutation, while polar motion is the corresponding variation with respect to the earth-fixed system. The third part of earth rotation is the variation in the length of the vector ω , i.e., the variation of the rotational velocity (length of the day) $\Omega = \|\omega\|$.

To obtain a parametrization of earth rotation where precession-nutation, polar motion and rotational velocity are separated, we introduce an intermediate moving geocentric Cartesian reference frame x', y', z' . The z' -axis direction coincides at any epoch with the direction of the instantaneous rotation axis, while the x' -axis direction is arbitrary. The transformation from the X, Y, Z (inertial) to the x', y', z' frame may be expressed in terms of three rotations:

$$R_3(\Theta_2) R_2(\Xi) R_1(H)$$

and the transformation from x', y', z' to the x, y, z (earth-fixed) frame can be similarly expressed as

$$R_1(-\eta) R_2(-\xi) R_3(\Theta_1)$$

Setting $\Theta = \Theta_1 + \Theta_2$, the total transformation from the inertial to the earth-fixed frame becomes

$$M = R_1(-\eta) R_2(-\xi) R_3(\Theta) R_2(\Xi) R_1(H) \quad (3-10)$$

where $\eta, \xi, \Theta, \Xi, H$ are all functions of time. (Note that the Θ_1 and Θ_2 components of Θ cannot be separated, and this justifies the arbitrariness of the x' -axis direction.) In order to introduce the instantaneous rotational velocity Ω in the model, noting that the $R_3(\Theta)$ rotation is about the instantaneous rotation axis (diurnal rotation), one may set

$$\Theta(t) = \Theta_0 + \int_{t_0}^t \Omega(\tau) d\tau \quad \Theta_0 = \Theta(t_0) \quad (3-11)$$

The three angles Ξ, H, Θ_0 represent the traditional precession-nutation, and the angles η and ξ the polar motion. The angle $(\Theta - \Theta_0)$ is similar to GAST (Greenwich Apparent Sidereal Time). The functions $\eta, \xi, \Omega, \Xi, H$ can now be approximated by step functions. To improve the approximation, we shall include in the transformation M the traditional precession-nutation based on current knowledge, as follows:

$$M = R_1(-\eta) R_2(-\xi) R_3(\Theta) R_2(\Xi) R_1(H) R^0 \quad (3-12)$$

where the known part of precession and nutation may be computed from [Mueller, 1969, Ch. 4].

$$R^0 = R_1(-\epsilon - \Delta\epsilon) R_3(-\Delta\psi) R_1(\epsilon) R_3(-z) R_2(\theta) R_3(-\zeta_0) \quad (3-13)$$

In the above expression, z , θ , ζ_0 are the Newcomb components of precession, and $\Delta\psi$ and $\Delta\epsilon$ the nutations in longitude and obliquity by Woolard. The transformation R^0 defines the traditional "true" frame of reference, an intermediate, noninertial, moving reference frame X' , Y' , Z' of known orientation, such that the Z' -axis is at any epoch near the instantaneous rotation axis. The angles Ξ , H , Θ_0 are thus small corrections to the reference precession-nutation parameters used in (3-13), and as such they may be considered as the transformation parameters between the "traditional (Newcomb/Woolard) true" frame and the "actual true" frame.

There are two alternatives in the step function representation of the angle Θ . One may represent Ω as a step function with constant value Ω_i over the i^{th} time interval (step), in which case

$$\Theta(t) = \Theta_0 + \sum_{k=1}^{i-1} \Omega_k (t_k - t_{k-1}) + \Omega_i (t - t_{i-1}) \quad (3-14)$$

$t_{i-1} < t \leq t_i$

A second choice is to approximate Θ over each step by a function of the form

$$\Theta_i(t) = \Theta_{0i} + \Omega_i (t - t_{i-1}) \quad t_{i-1} \leq t < t_i \quad (3-15)$$

This is a weaker representation from the adjustment point of view because of the larger number of parameters (Θ_{0i} , $i = 1, 2, \dots, N$ vs. Θ_0), but it leads to estimates of variances of the estimates of the step function constants η_i , ξ_i , Θ_{0i} , Ω_i , Ξ_i , H_i , which are uniform (equal) over all steps (t_{i-1} , t_i). On the contrary, the first representation leads to variances which are smaller for steps in the middle and larger for steps at the ends of the original time span of the available observations.

Over each step the model for the VLBI observations becomes

$$d_{ij,k} = b_{ij}^T R_1(-\eta) R_2(-\xi) R_3[\Theta_0 + \Omega(t_k - t_0)] R_2(\Xi) R_1(H) R^0 e_p + c[C_{0ij} + C_{ij}(t_k - t_0)] \quad (3-16)$$

where η , ξ , Θ_0 , Ω , Ξ , H are constants over the considered step (the subscripts identifying the steps have been dropped for simplicity), t_k is the epoch of observation, and t_0 is the beginning of the step containing t_k .

4. Linearization of Observation Equations

For the time span of available observations considered in this study (maximum of one month), the effect of precession-nutation on the experimental design (relative radio sources - stations configuration) is negligible. One can therefore drop the known transformation matrix R^0 from the model without any significant change on the simulation results. This simplifies somewhat the equations following, but it must be understood that appropriate modifications must be made when analyzing real data by including the effect of R^0 .

The angles ξ , η , Ξ , H are very small since both the z and Z axes (actually the noninertial Z' -axis on account of ignoring R^0) are or can be chosen to be near the rotation axis. The usual approximations, $\cos \alpha = 1$, $\sin \alpha = \alpha$, $\alpha^2 = 0$, for a small angle α may be used, leading to

$$R_1(-\eta) R_2(-\xi) = \begin{pmatrix} 1 & 0 & \xi \\ 0 & 1 & -\eta \\ -\xi & \eta & 1 \end{pmatrix}$$

and

$$R_2(\Xi) R_1(H) = \begin{pmatrix} 1 & 0 & -\Xi \\ 0 & 1 & H \\ \Xi & -H & 1 \end{pmatrix}$$

The model now becomes

$$\begin{aligned} d_{1jpk} &= (r_j - r_i)^T S^T(\xi, \eta) R_3(\Theta_0 + \Omega \tau_k) S(\Xi, H) e_p + \\ &+ c [C_{\alpha 1j} + C_{1j} \tau_k] \end{aligned} \quad (4-1)$$

where $\tau_k = t_k - t_0$; $e_p = [\cos \delta_p \cos \alpha_p, \cos \delta_p \sin \alpha_p, \sin \delta_p]^T$, α_p and δ_p are the right-ascension and declination of the p^{th} radio source with respect to the X, Y, Z inertial frame; $r_i = [x_i, y_i, z_i]^T$, x_i, y_i, z_i are the coordinates of the i^{th} station with respect to the x, y, z earth-fixed frame; and

$$S(a, b) = \begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & b \\ a & -b & 1 \end{pmatrix} \quad (4-2)$$

ORIGINAL PAGE IS
OF POOR QUALITY.

The available observations d_{ijpk} form a vector of observations L_b , corresponding to a vector of observables $L_a = F(X_a)$, where X_a is a vector of unknown parameters. The unknowns are the station coordinates $x_1, y_1, z_1, i = 1, 2, \dots, N_s$, the earth rotation parameters $\xi, \eta, \Theta_0, \Omega, \Xi, H$ (one set of six per each step), radio source coordinates $\alpha_p, \delta_p, p = 1, 2, \dots, N_{rs}$, and clock synchronization parameters $C_{oij}, C_{tj}, i, j = 1, 2, \dots, N_s$.

The d_{ijpk} components of the vector L_a are linear with respect to most of the parameters with the exception of $\Theta_0, \Omega, \alpha_p$, and δ_p . A completely linearized model can be obtained by means of approximate values X^0 of the parameters X_a . Expansion in Taylor series and neglect of second- and higher-order terms leads to

$$L_a = F(X^0) + \left. \frac{\partial F(X_a)}{\partial X_a} \right|_{X^0} (X_a - X^0) \quad (4-3)$$

The actual observations L_b differ from the observables L_a because of the presence of unknown observations errors V , according to $L_a = L_b + V$. Setting $A = (\partial F / \partial X_a) \big|_{X^0}$, $L^0 = F(X^0)$, $X = X_a - X^0$, and $L = L^0 - L_b$, the linearized observation equations are obtained:

$$V = A X + L \quad (4-4)$$

A and L are known, while X and V are unknown. The vector V is assumed to be an outcome of a random vector with $E[V] = 0$ and known covariance matrix $E[V V^T] = S$.

Application of standard least squares techniques leads to a solution X , minimizing the quadratic form $V^T P V$ where $P = S^{-1}$, which satisfies the "normal equations,"

$$(A^T P A) X = -A^T P L \quad \text{or} \quad N X = U \quad (4-5)$$

To compute the elements of the design matrix A , the partial derivatives of d_{ijpk} with respect to the parameters are required. Introducing the notation, $\psi_k = \Theta_0 + \Omega \tau_k$, $\chi_{kp} = \psi_k - \alpha_p$, $\hat{x}_{ij} = x_j - x_1$, $y_{ij} = y_j - y_1$, $z_{ij} = z_j - z_1$, and dropping subscripts where confusion is not likely to arise, the partial derivatives are as follows:

$$\frac{\partial d}{\partial x_j} = - \frac{\partial d}{\partial x_1} = \cos \delta \cos \chi + \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi)$$

$$\frac{\partial d}{\partial y_j} = - \frac{\partial d}{\partial y_1} = -\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)$$

$$\frac{\partial d}{\partial z_j} = - \frac{\partial d}{\partial z_1} = \sin \delta - \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)$$

$$\begin{aligned} \frac{\partial d}{\partial \xi} = & x_{1j} [\sin \delta + \cos \delta (\Xi \cos \alpha - H \sin \alpha - \xi \cos \chi)] + \\ & + y_{1j} [\eta \cos \delta \cos \chi] + \\ & + z_{1j} [-\cos \delta \cos \chi - \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi)] \end{aligned}$$

$$\begin{aligned} \frac{\partial d}{\partial \eta} = & y_{1j} [-\sin \delta + \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)] + \\ & + z_{1j} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)] \end{aligned}$$

$$\begin{aligned} \frac{\partial d}{\partial \Theta_0} = & x_{1j} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi)] + \\ & + y_{1j} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] + \\ & + z_{1j} [\cos \delta (\xi \sin \chi - \eta \cos \chi)] \end{aligned}$$

$$\frac{\partial d}{\partial \Omega} = \tau_k \frac{\partial d}{\partial \Theta_0}, \quad \frac{\partial d}{\partial C_{01j}} = c, \quad \frac{\partial d}{\partial C_{1j}} = c \tau_k$$

$$\begin{aligned} \frac{\partial d}{\partial \Xi} = & x_{1j} \left\{ -\cos \psi \cos \delta + \cos \delta [\xi \cos \alpha + \cos \psi (-\Xi \cos \alpha + H \sin \alpha)] \right\} + \\ & + y_{1j} \left\{ \sin \psi \sin \delta + \cos \delta [-\eta \cos \alpha + \sin \psi (\Xi \cos \alpha - H \sin \alpha)] \right\} + \\ & + z_{1j} [\cos \delta \cos \alpha + \sin \delta (\xi \cos \psi + \eta \sin \psi - \Xi)] \end{aligned}$$

$$\begin{aligned} \frac{\partial d}{\partial H} = & x_{1j} [\sin \delta \sin \psi + \cos \delta \sin \alpha (\Xi \cos \psi - H \sin \psi - \xi)] + \\ & + y_{1j} [\sin \delta \cos \psi + \cos \delta \sin \alpha (-\Xi \sin \psi - H \cos \psi + \eta)] + \\ & + z_{1j} [-\cos \delta \sin \alpha + \sin \delta (-\xi \sin \psi + \eta \cos \psi - H)] \end{aligned}$$

$$\begin{aligned} \frac{\partial d}{\partial \alpha} = & x_{1j} [\cos \delta \sin \chi] + y_{1j} [\cos \delta \cos \chi] + \\ & + z_{1j} \cos \delta (-\xi \sin \chi + \eta \cos \chi - \Xi \sin \alpha - H \cos \alpha) \end{aligned}$$

$$\begin{aligned}
\frac{\partial d}{\partial \delta} = & x_{1j} [-\sin \delta \cos \chi + \cos \delta (-\Xi \cos \psi + H \sin \psi + \xi)] + \\
& + y_{1j} [\sin \delta \sin \chi + \cos \delta (\Xi \sin \psi + H \cos \psi - \eta)] + \\
& + z_{1j} [\cos \delta + \sin \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)]
\end{aligned} \tag{4-6}$$

A second type of possible observations is the time derivative of d_{1jpk}

$$f_{1jpk} = \frac{\partial d_{1jpk}}{\partial t} = \Omega (r_j - r_1)^T S^T(\xi, \eta) P_3 R_3(\Theta_0 + \Omega \tau_k) S(\Xi, H) e_p + c C_{1j} \tag{4-7}$$

where

$$P_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4-8}$$

The related partial derivatives of f_{1jpk} with respect to the parameters are as follows:

$$\begin{aligned}
\frac{1}{\Omega} \frac{\partial f}{\partial \xi} = & x_{1j} [\xi \cos \delta \sin \chi] - y_{1j} [\eta \cos \delta \sin \chi] \\
& + z_{1j} [\cos \delta \sin \chi - \sin \delta (\Xi \sin \psi + H \cos \psi)]
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\Omega} \frac{\partial f}{\partial \eta} = & y_{1j} [\cos \delta (-\xi \sin \chi + \eta \cos \chi)] \\
& + z_{1j} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)]
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\Omega} \frac{\partial f}{\partial \Xi} = & x_{1j} [\sin \delta \sin \psi - \cos \delta \sin \psi (-\Xi \cos \alpha + H \sin \alpha)] \\
& + y_{1j} [\sin \delta \cos \psi - \cos \delta \cos \psi (-\Xi \cos \alpha + H \sin \alpha)] \\
& + z_{1j} [\sin \delta (-\xi \sin \psi + \eta \cos \psi)]
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\Omega} \frac{\partial f}{\partial H} = & x_{1j} [\cos \psi \sin \delta - \cos \delta \sin \alpha (\Xi \sin \psi + H \cos \psi)] \\
& + y_{1j} [-\sin \psi \sin \delta + \cos \delta \sin \alpha (-\Xi \cos \psi + H \sin \psi)] \\
& + z_{1j} [\sin \delta (-\xi \cos \psi - \eta \sin \psi)]
\end{aligned}$$

$$\begin{aligned}\frac{1}{\Omega} \frac{\partial f}{\partial \Theta_o} = & x_{1j} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] \\ & + y_{1j} [\cos \delta \sin \chi - \sin \delta (-\Xi \sin \psi - H \cos \psi)] \\ & + z_{1j} [\cos \delta (\xi \cos \chi + \eta \sin \chi)]\end{aligned}$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial x_j} = -\frac{1}{\Omega} \frac{\partial f}{\partial x_1} = -\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi)$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial y_j} = -\frac{1}{\Omega} \frac{\partial f}{\partial y_1} = -\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial z_j} = -\frac{1}{\Omega} \frac{\partial f}{\partial z_1} = \cos \delta (\xi \sin \chi - \eta \cos \chi)$$

$$\begin{aligned}\frac{1}{\Omega} \frac{\partial f}{\partial \alpha} = & x_{1j} [\cos \delta \cos \chi] + y_{1j} [-\cos \delta \sin \chi] \\ & + z_{1j} [\cos \delta (-\xi \cos \chi - \eta \sin \chi)]\end{aligned}$$

$$\begin{aligned}\frac{1}{\Omega} \frac{\partial f}{\partial \delta} = & x_{1j} [\sin \delta \sin \chi + \cos \delta (\Xi \sin \psi + H \cos \psi)] \\ & + y_{1j} [\sin \delta \cos \chi + \cos \delta (\Xi \cos \psi - H \sin \psi)] \\ & + z_{1j} [\sin \delta (-\xi \sin \chi + \eta \cos \chi)]\end{aligned}$$

$$\frac{\partial f}{\partial \Omega} = \tau_k \frac{\partial f}{\partial \Theta_o}, \quad \frac{\partial f}{\partial C_{oij}} = 0, \quad \frac{\partial f}{\partial C_{ij}} = c \quad (4-9)$$

5. Coordinate System Definition and Inner Constraints

The combination of all observations leads to the linearized observation equations

$${}_nV_1 = {}_nA_u X_1 + {}_nL_1 \quad (5-1)$$

Minimization of $V^T P V$ leads to a solution satisfying the normal equations [Uotila, 1967]

$$N X = U \quad (5-2)$$

where

$$N = A^T P A \quad \text{and} \quad U = -A^T P L.$$

Because of the lack of coordinate system definition, N is singular with rank $(N) = r < u$, where u is the number of parameters and $s = u - r$ is the rank deficiency of N .

Originally $s = 9$, corresponding to the six degrees of freedom in the definition of the earth-fixed coordinate system (three for origin position and three for orientation) plus the three degrees of freedom in the inertial system definition (orientation only)

Since a radio source catalogue of a certain accuracy is assumed to be available, radio source coordinates are treated as observations (weighted unknowns) rather than as unknown parameters. The inertial frame is therefore defined through the catalogued radio source coordinates and the remaining rank deficiency of N becomes $s = 6$. A unique solution to the normal equations may be obtained if in addition a set of minimal linear constraints are imposed on the parameters, of the form

$$C^T X = 0 \quad (5-3)$$

where C is a $u \times s$ matrix and $\text{rank}(C) = s$.

Among the various possible solutions to the normal equations, the unique one given by $X = N^+ U$, where N^+ is the pseudoinverse of N , has the following properties [Blaha, 1971]: $X^T X = \min.$ and $\text{trace } N^+ = \min.$ In view of the interpretation of N^+ as the variance-covariance matrix of the parameters, the second property makes the solution optimal in the sense that smaller variances of the parameters provide a better representation of parameter related estimable quantities in terms of the

nonestimable parameters. To avoid the use of pseudoinverse computation algorithm, one may resort to a particular set of minimal constraints called inner constraints [Blaha, 1971],

$$E^T X = 0 \quad (5-4)$$

which leads to the same solution for X as the one obtained with the use of the pseudoinverse N^+ . It can be shown that such a set of inner constraints can be obtained with the help of a $u \times s$ matrix E ($\text{rank}(E) = s$) satisfying

$$N E = 0 \quad (5-5)$$

An algebraic type of proof can be found in [Pope, 1971] which settles the truth of the matter but throws little light on the interrelation between inner constraints and pseudoinverse. A different type of proof is given in Appendix A of this work, based on the geometry of the operator represented by the matrix N .

In view of the fact that $A E = 0 \Rightarrow N E = A^T P A E = 0$, a set of inner constraints has been analytically constructed using six independent solutions to the set of equations

$$A^j y = 0 \quad j = 1, 2, \dots, u \quad (5-6)$$

where A^j denotes the j^{th} row of A . We assume that the approximate values of η , ξ , Ξ , H are zero in the computation of partials of observations with respect to the unknowns, thus obtaining simpler analytical expressions for the elements of A . Letting the order of the unknowns in X to be the following

$$X^T = [d\eta, d\xi, d\Theta_0, d\Omega, dx_1, dy_1, dz_1, \dots, dx_N, dy_N, dz_N]$$

one may set

$$y^T = [w_1, w_2, w_3, w_4, \alpha_1, \beta_1, \gamma_1, \dots, \alpha_N, \beta_N, \gamma_N]$$

For a row of A corresponding to a d_{ijk} observation, one has

$$\begin{aligned} w_1 \frac{\partial d}{\partial \eta} + w_2 \frac{\partial d}{\partial \xi} + w_3 \frac{\partial d}{\partial \Theta_0} + w_4 \frac{\partial d}{\partial \Omega} + \alpha_1 \frac{\partial d}{\partial x_1} + \beta_1 \frac{\partial d}{\partial y_1} + \gamma_1 \frac{\partial d}{\partial z_1} + \\ + \alpha_j \frac{\partial d}{\partial x_j} + \beta_j \frac{\partial d}{\partial y_j} + \gamma_j \frac{\partial d}{\partial z_j} = 0 \end{aligned} \quad (5-7)$$

With the analytical expressions of the partials and after a considerable algebraic effort, one arrives at six independent solutions y_i , $i = 1, 2, \dots, 6$, and the inner constraint matrix E is formulated by setting

$$E = [y_1 \ y_2 \ \dots \ y_6]$$

The final result is

$$\begin{array}{cccccccccc} d\eta & d\eta & d\Theta_o & d\Omega & dx_1 & dy_1 & dz_1 & \dots & dx_N & dy_N & dz_N \\ \left[\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & -z_1^o & y_1^o & \dots & 0 & -z_N^o & y_N^o \\ 0 & 1 & 0 & 0 & z_1^o & 0 & -x_1^o & & z_N^o & 0 & -x_N^o \\ 0 & 0 & -1 & 0 & -y_1^o & x_1^o & 0 & & -y_N^o & x_N^o & 0 \end{array} \right] \end{array} \quad (5-8)$$

where x_1^o , y_1^o , z_1^o are the approximate values of station coordinates used in the computation of the design matrix A . More explicitly there are two sets of constraints. The first one

$$\sum_i \begin{pmatrix} dx_i \\ dy_i \\ dz_i \end{pmatrix} = 0 \quad (5-9)$$

"defines" the origin of the earth-fixed system, while the second one

$$\begin{pmatrix} -d\eta \\ -d\xi \\ d\Theta_o \end{pmatrix} = \sum_i \begin{pmatrix} 0 & -z_i^o & y_i^o \\ z_i^o & 0 & -x_i^o \\ -y_i^o & x_i^o & 0 \end{pmatrix} \begin{pmatrix} dx_i \\ dy_i \\ dz_i \end{pmatrix} \quad (5-10)$$

"defines" its orientation.

In the discussion above, the problem of epoch and time scale definition for the set of station clocks has been ignored. The parameters C_{o1j} and C_{1j} refer only to relative offsets and drifts between the clocks at stations i and j .

Usually epoch and time scale are provided by the readings of one of the clocks (master clock), and they are transformed to other station clocks through the parameters C_{mj} , C_{nj} where the master clock is at the m^{th} station.

Another way to provide epoch and time scale is the use of inner constraints defining a fictitious master clock. We may shift to a new set of parameters by setting

$$C_{ojj} = C_{oj} - C_{op1} \quad C_{ij} = C_{pj} - C_{p1} \quad (5-11)$$

The new parameters are offsets C_{op1} and drifts C_{p1} with respect to the fictitious master clock, and the additional inner constraints are

$$\sum_i dC_{op1} = 0 \quad \text{and} \quad \sum_i dC_{p1} = 0 \quad (5-12)$$

6. Remarks Related to Design of Experiment Concepts

Since the usual objective of a geodetic experiment is the estimation of certain parameters, an experimental design may be considered optimal when the errors in parameter estimates are minimized in some certain sense. Within the linear least squares model, the a posteriori variance-covariance matrix of parameter estimates is given in general by

$$S_x = \sigma_0^2 (A^T P A)^+ = \sigma_0^2 Q^+ \quad (6-1)$$

where Q^+ denotes the pseudoinverse of a square matrix Q , P is the weight matrix and σ_0^2 the a posteriori estimate of the variance of unit weight. The "errors" in parameter estimates therefore depend on the observational accuracy and on the design matrix A . The design matrix $A = A(X^0)$ itself depends on the approximate values of parameters X^0 . The determination of optimal X^0 values minimizing some risk function (e g., trace S_x) is usually referred to as the "configuration problem," or the "first-order optimal design" [Grafarend, 1974]. On the assumption that the risk function chosen is insensitive to small changes of X^0 , the approximate values of parameters may be identified with the true ones. In a similar way the small

angles η , ξ , Ξ , H in our model may be considered to be zero. Of the remaining parameters Θ_0 depends on the choice of reference frames, Ω is approximated by the fixed average rotational velocity of the earth, while in the case of transcontinental baselines the choice of radio source coordinates α_p , δ_p is strongly restricted by observability conditions. Therefore, the first-order optimal design is practically limited to the determination of the optimal configuration of the network of observing stations.

The order of magnitude of elements in a row of the design matrix A determines the sensitivity of the corresponding observation with respect to each of the parameters. Next a short geometric interpretation of these partials is given to provide possible "hints" towards the determination of the optimal design.

The partial derivatives with respect to η , ξ , Θ_0 , Ω , Ξ , H (earth rotation parameters) and α_p , δ_p are proportional to the baseline length. Indeed, they all contain linear terms in $x_{ij} = r_{ij} x_{ij}^0$, $y_{ij} = r_{ij} y_{ij}^0$ and $z_{ij} = r_{ij} z_{ij}^0$, where r_{ij} is the length of the ij baseline and x_{ij}^0 , y_{ij}^0 , z_{ij}^0 are the components of the unit vector r_{ij}^0 in the direction of the baseline. It is therefore possible to define "sensitivity per unit of baseline length" by means of $\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta}$ where β stands for any of the earth rotation or radio source coordinates parameters. It can be shown that

$$\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} = (r_{ij}^0)^T S_\beta \quad (6-2)$$

i.e., that the sensitivity per unit of baseline length is the projection of some vector S_β (called the parameter β sensitivity vector) on the direction of the baseline. It can be easily verified that the parameter sensitivity vectors are given by.

$$\begin{aligned} S_\eta &= -R_1(\pi/2) P_{yz} R_3(\chi) e_6 \\ S_\xi &= -R_2(\pi/2) P_{xz} R_3(\chi) e_6 \\ S_{\Theta_0} &= \tau_k^{-1} S_\Omega = P_{xy} R_3(\chi + \pi/2) e_6 \\ S_\Xi &= R_3(\psi) R_2(\pi/2) P_{xz} R_3(-\alpha) e_6 \\ S_H &= R_3(\psi) R_1(\pi/2) P_{yz} R_3(-\alpha) e_6 \\ S_\alpha &= -R_3(\chi + \pi/2) P_{xy} e_6 \\ S_\delta &= R_3(\chi) R_2(\pi/2) e_6 \end{aligned} \quad (6-3)$$

where

$$\mathbf{e}_\delta = [\cos \delta, 0, \sin \delta]^T, \quad \mathbf{e}_p = R_3(-\alpha) \mathbf{e}_\delta, \text{ and}$$

$$\mathbf{P}_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{P}_{yz} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{P}_{xz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are projection matrices on the xy, yz and xz planes, respectively. The position of the sensitivity vectors varies with time, and their loci, all contained within the unit sphere, are depicted in Fig. 6.1. Maximum sensitivity occurs when the baseline is parallel to the corresponding parameter sensitivity vector, while an observation is insensitive to a parameter when the baseline and parameter sensitivity vector are perpendicular. With this in mind and with the help of Fig. 6.1, Table 6.1 is constructed, summarizing the sensitivity to various parameters for extreme cases of baseline and radio source directions.

Table 6.1
Sensitivity of Baseline - Radio Source Extreme Configurations
with Respect to Various Parameters

Parameters	Baseline Parallel to Equator ($z_{1j} = 0$)		Baseline Parallel to Rotation Axis ($x_{1j} = y_{1j} = 0$)	
	Equatorial Radio Sources ($\delta = 0^\circ$)	Polar Radio Sources ($\delta = 90^\circ$)	Equatorial Radio Sources ($\delta = 0^\circ$)	Polar Radio Sources ($\delta = 90^\circ$)
ξ, η	no	yes	yes	no
Ξ, H	no	yes	yes	no
Θ_o, Ω, α	yes	no	no	no
δ	no	yes	yes	no

Note that observations in the fourth column are impossible due to observability conditions (a baseline parallel to the rotation axis must be located near the equator and observe equatorial radio sources only).

The sensitivity with respect to Ω as well as to C_{1j} is proportional to the time span τ_k of the available observations while that to C_{1j} and C_{o1j} is independent of the baseline - radio source configuration.

Sensitivity with respect to station coordinates is independent of baseline length. If e_x , e_y , e_z denote unit vectors in the directions of the x, y, z axes, respectively, the relevant partial derivatives under the same approximations are

$$\frac{\partial d}{\partial x_j} = e_x^T S_c, \quad \frac{\partial d}{\partial y_j} = e_y^T S_c, \quad \frac{\partial d}{\partial z_j} = e_z^T S_c$$

where $S_c = R_3(-\chi) e_6$. The sensitivity of observations with respect to station coordinates is determined by the projection of the coordinate sensitivity vector S_c on the coordinate axes. The locus of S_c is depicted in Fig. 6.1. It can readily be seen that observations to equatorial radio sources are insensitive to the z station coordinates, while observations to polar radio sources are insensitive to x and y station coordinates.

All the above remarks are of a rather general nature and simply provide "hints" towards the design of an optimal station configuration for the VLBI experiment, which also depends on which of the parameters in particular is to be determined.

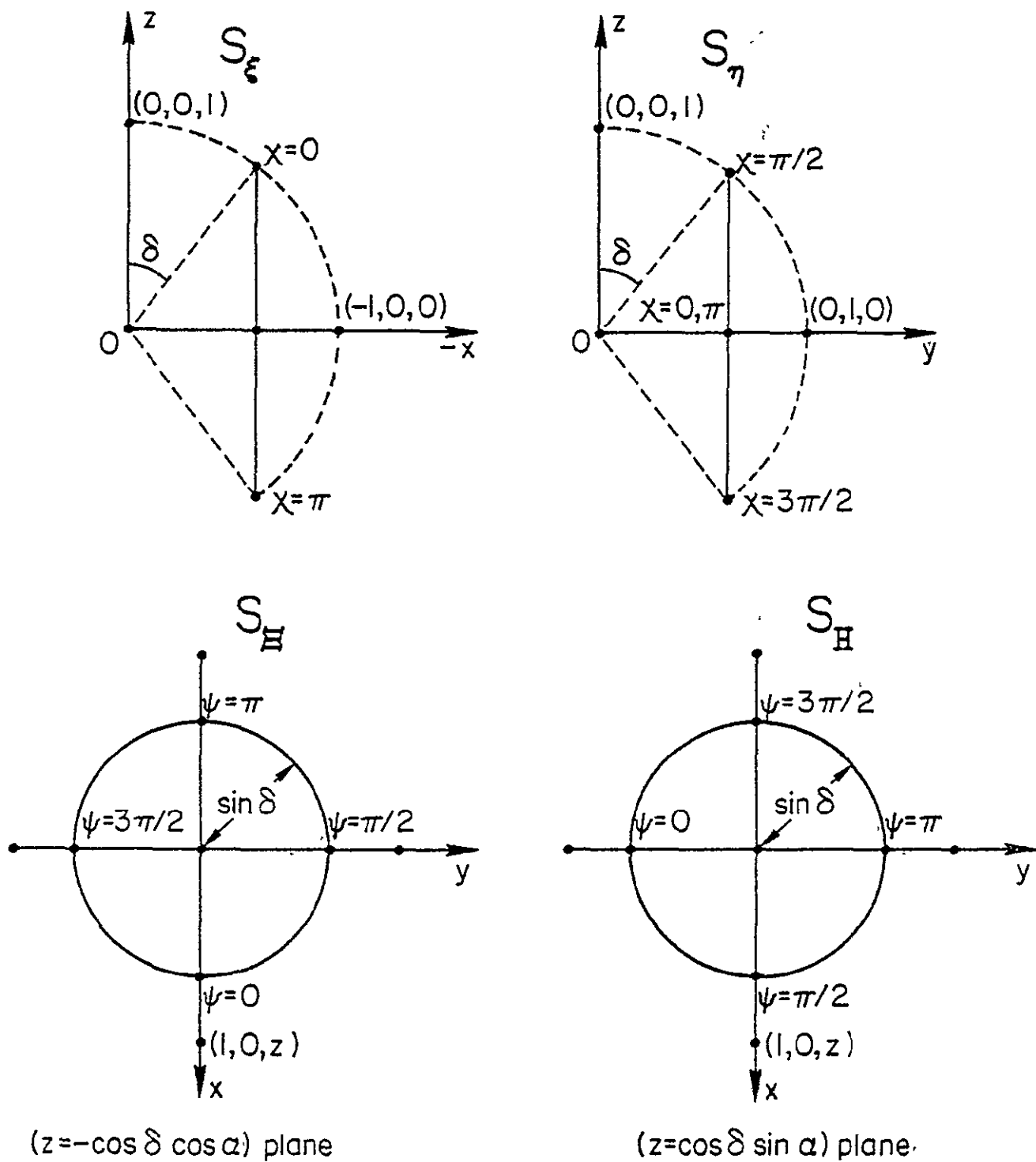


Fig. 6.1 Geometric loci of parameter sensitivity vectors.

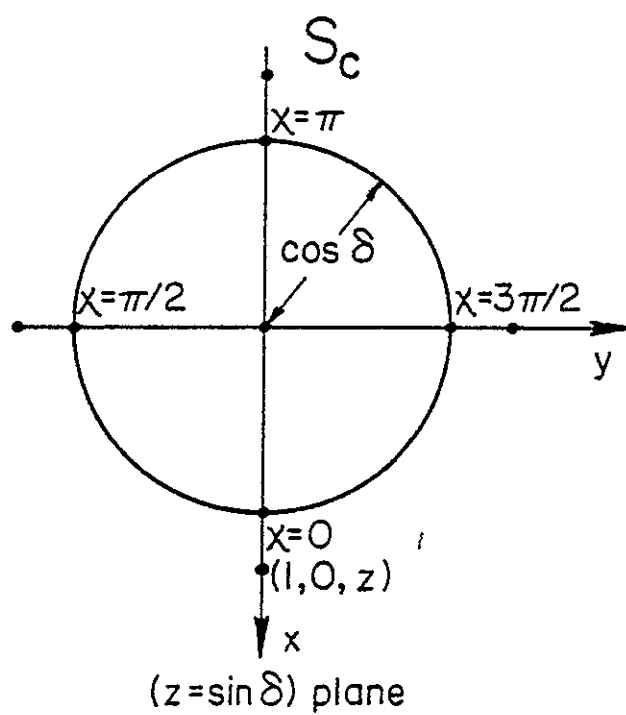
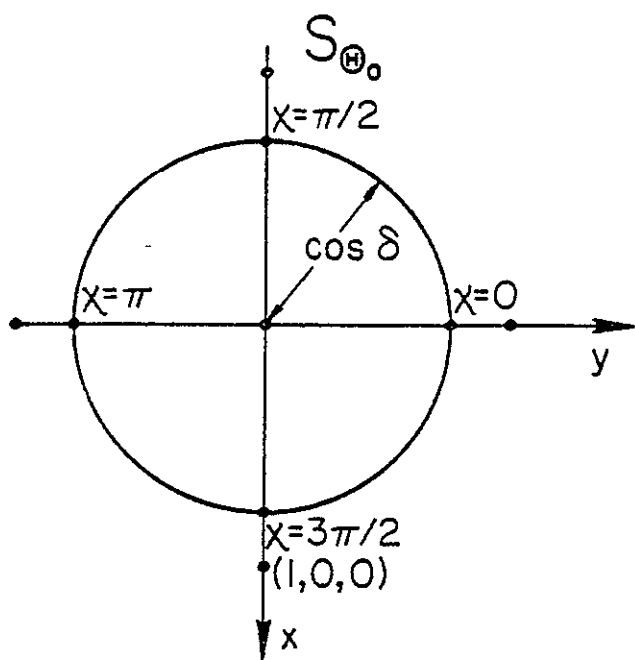
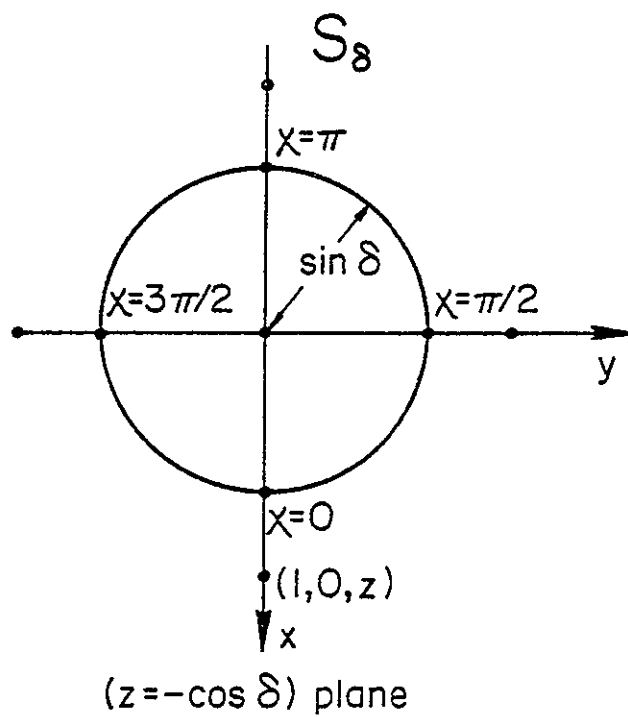
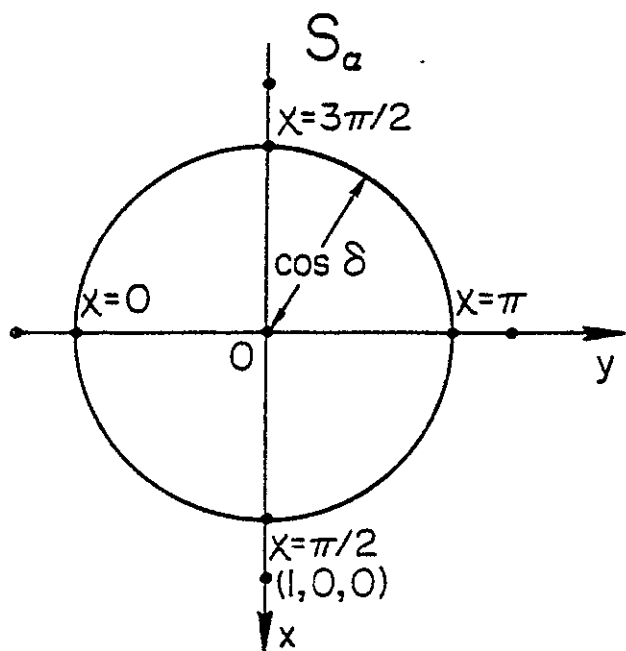


Fig. 6.1 (cont'd)

7. Simulation and Adjustment Philosophy

The objective of this simulation is the determination of standard deviations of parameter estimates relevant to VLBI observations for a number of characteristic network designs. In addition to this design (relative configuration of station - radio sources and arrangement of observations in time), the a posteriori standard deviations of parameters also depend on a number of other factors. These include the accuracy of observations, their total time span, and the modeling of earth rotation and relative station motions in the adjustment. Since step function models are used here, the time interval over which earth rotation parameters are considered to be constant ("earth step") and the time interval over which station coordinates are considered constants ("station step") become also variables. It has also been assumed that a radio source coordinate catalogue is available, and its accuracy is one additional variable affecting final results. Summarizing, the problem is to examine the variation of standard deviations of the following parameters:

- i. station coordinates
- ii. earth rotation parameters

with respect to the following independent variables:

- a. observational noise
- b. time span of observations
- c. earth step
- d. station step
- e. accuracy of radio source catalogue
- f. station - radio source configuration, arrangement of observations in time.

With respect to the last item above, three different variables appear in the same group. This is justified in view of the fact that once a transcontinental network design has been decided upon, observability conditions impose strong limitations on both the choice of radio sources to be observed and the arrangement of observations in time.

The fact that observations are not instantaneous, but averages over some time interval (integration time of 7-10 minutes), imposes also limitations on the maximum density of observations in time.

The choice of earth and station step in real data analysis depends on the frequency content of the corresponding time functions. In view of the aliasing effect [Koopmans, 1974], the smaller the step size, the higher the spectral resolution of the corresponding functions. On the other hand, larger step size leads to a smaller number of parameters to be estimated and a "stronger" adjustment with more degrees of freedom. The choice of optimal step size in the analysis of real data constitutes a very interesting problem worthy of a separate study.

In the present simulation study, step size has been considered as a variable, while the corresponding functions generated in the simulation of observations are continuous rather than step functions. However, precautions have been taken so that the total variation $V_{f,I}$ of the function $f(t)$ in question over each step time interval I is insignificant compared to observational noise. In fact,

$$V_{f,I_i} = \max_{t \in I_i} [f(t)] - \min_{t \in I_i} [f(t)] \leq \sigma_b/N \quad i = 1, 2, \dots, n$$

where σ_b is the standard deviation of the observations, and N is a large integer (specifically here $N = 10$).

In view of a 7-10-minute integration time, and allowing for antenna motion, observations (time delays and their time derivatives) are uniformly spaced at 15-minute intervals over the total time span considered. Their corresponding standard deviations σ_b and σ_b' are taken to be of a constant ratio $\sigma_b/\sigma_b' = 1/2$, where σ_b is in nsec and σ_b' in nsec/hr. This value is typical, but the ratio may be somewhat different depending on antenna and observed signal characteristics. The choice of a fixed ratio reduces the number of considered parameters by one with a corresponding significant reduction in computational effort.

Once a certain observational pattern has been decided upon, a simulation program (VLBI SIMULATOR) produces perfect observations using the simplified model (Eq. 4-1). This program requires as input station and radio source true coordinates and the pattern of the observations (baseline - radio source - epoch of

observation). The station coordinates are taken to be constants, while the values of earth rotation parameters at the epoch of observation are provided by a subroutine (SUBROUTINE EARTH). Θ_0 is fixed to a realistic value for the beginning of the observations t_0 . The effect of traditional precession-nutation is absent from the simulation as well as from the adjustment model for computational simplicity. The inertial frame thus is taken to be the true equatorial system of the epoch t_0 so that its Z-axis is near the rotational axis. For the same reason, the earth-fixed frame is taken to be the CIO/BIH zero-meridian terrestrial system.

The perfect observations are processed through a program which adds "Observational errors" drawn from a Gaussian population with zero mean and desired variance.

For the adjustment of the observations, two separate programs are used. The first one (PROG01) processes observations of total time span equal to both earth and station step (one day). An option is available for processing any desired number of different radio source catalogue accuracies in the same run.

The second program (PROG02) processes observations of time span equal to the station step only, which in turn is taken to be an integer multiple of the earth step. It is, however, possible to process observations using unequal earth steps within the total span of available observations. Within each earth step the linearized observations equations are of the form

$$V_1 = \dot{A}_1 \dot{X} + \ddot{A}_1 \ddot{X}_1 + L_1$$

where \dot{X} refers to parameters common to all earth steps (station and radio source coordinates) and \ddot{X}_1 refers to parameters particular to each earth step (earth rotation parameters).

The total set of such individual observation equations is combined in a unified solution through a "second-order partitioned linear regression" scheme, extended to incorporate inner constraints. The detailed algorithm is presented in Appendix B. The related programs and subroutines are in Appendix C.

8. Arrangement of Experiments

To obtain more realistic results, station locations considered here are not completely arbitrary, but limited to present and prospective locations of VLBI stations. Such candidate station locations are presented in Table 8.1. In the same table the stations actually used in this study are indicated by asterisks. Two criteria have been considered in the selection. First, stations close to plate boundaries, i.e., in geophysically active areas, have been excluded to avoid possible comparatively large station drifts deviating from those considered in our simulations. Secondly, from groups of stations close to each other, only one station has been included on the assumption that final results would be similar for a nearby station. It is recognized that these criteria probably exclude the most active stations currently in operation, but this should not influence the general conclusions sought.

An experiment consists of a group of stations simultaneously participating in the observations. In Table 8.3 the station groups for the experiments considered in this study are presented. After a certain station group has been selected, the arrangement of observations is one observations every 15 minutes, which interval and choice of the radio sources are limited by observability conditions. The observability condition naturally imposed is that the observed radio source must be above the horizon at both ends of the observing baseline. To avoid large atmospheric refraction errors, the zenith distance of the radio source is limited, in general, to be less than $z_{\max} = 45^\circ$, with some exceptional cases where $z_{\max} = 60^\circ$.

Such observability conditions limit the observable part of the sky to within the intersection of two cones with the local station verticals as axes and z_{\max} as the vertex angle. For transcontinental baselines, the region of observability is small and the choice is between observing a certain quasar or a neighboring one without significant effect on the experimental design.

In setting out a certain sequence of observations, the choice between possible alternatives has been guided by the hints obtained in Chapter 6, with emphasis on sensitivity with respect to earth motion and station coordinate parameters.

Table 8.1
Present and Prospective VLBI Station Locations and
Their Approximate Coordinates

No.	Location	Longitude	Latitude
	Kashima, Japan	137°	35°
1	Canberra, Australia *	149	-35
2	Kauai, Hawaii *	200	22
3	Fairbanks, Alaska *	212	65
	Goldstone, California	243	35
	Algonquin, Canada	277	51
	Greenbank, Virginia	282	38
4	Haystack, Massachusetts *	288	41
	Santiago, Chile	289	-34
	Arecibo, Puerto Rico	294	18
5	Sao Paulo, Brazil *	313	-24
6	Madrid, Spain *	356	40
	Bonn, Germany	7	51
7	Onsala, Sweden *	17	59
8	Johannesburg, So. Africa *	28	-26
	Crimea, USSR	34	45

* indicates stations considered in this study

Table 8.2

Fictitious Stations Used in Experiments 6 and 9

No.	Location	Longitude	Latitude
9	equatorial	0°	0°
10	"	60	0
11	"	120	0
12	"	180	0
13	"	240	0
14	"	300	0
15	meridian	0	0
16	"	0	60
17	"	180	60
18	"	180	0
19	"	180	-60
20	"	0	-60

Table 8.3

Experiments and Participating Baselines
(Station Nos. Refer to Numbering in Table 8.1 or 8.2)

Experiment No.	Participating Baselines
1	2-3, 2-4, 3-4
2	3-4, 3-7, 4-7
3	4-5, 4-6, 5-6
4	5-6, 5-8, 6-8
5	1-2, 2-3, 2-4, 3-4, 3-7, 4-5, 4-6, 4-7, 5-6, 5-8, 6-7, 6-8
6	9-10, 10-11, 11-12, 13-14, 14-9
7	2-4, 4-5
8	2-4, 4-6
9	15-16, 16-17, 17-18, 18-19, 19-20, 20-15
10	3-4, 4-7

An effort has also been made to observe radio sources of a variety of declinations to avoid the critical configuration appearing when, for example, all radio sources are of the same declination. From a larger set of available radio sources, only a rather uniformly distributed set has been considered. The criterion for inclusion has been the use of the radio source in previous experiments so that its appropriateness for VLBI observations is assured.

Since a radio source catalogue of a certain accuracy is assumed to be available, no effort has been made to optimize radio source coordinate recovery. As a result, this study is of no "astrometric" value, and no significant improvement of radio source coordinate accuracy can be found in the results of the adjustment performed.

One sidereal day can be taken as the "unit" for an observational pattern since a sequence of observations performed over one day can be repeated over the next days. For observations of a total time span of a number of days, the same daily pattern is repeated in the simulation. Various experimental designs are compared to each other on the basis of one day of observations where earth rotation and baseline parameters are considered as constants over the whole day.

The choice of experiments performed has been primarily directed towards the possibility of recovering earth rotation parameters from a minimum number of observing stations. The minimum number is the three stations necessary for the definition of an earth-fixed system through minimal or inner constraints. Experiments 1, 2, 3 and 4 are cases of such minimal "triangle networks" where all the sides of the triangle are observing baselines.

A weaker design (Experiments 7, 8, 10) is a three station configuration where only two of the triangle sides are observing baselines. In particular, Experiment 10 is a counterpart of Experiment 2, such that the effect of removing one baseline (other design parameters remaining the same) can be studied. The opposite to such minimal three-station designs is provided in Experiment 5 where all eight stations participate in the observations.

In addition to these "realistic" station locations, two "fictitious" designs are also considered, their common characteristic being the closure of a network of stations on a great circle around the earth. Experiment 6 is such a network of uniformly spaced stations along the equator, while Experiment 9 is of a similar design along a meridian. The coordinates of stations involved in these experiments are listed in Table 8.2.

Fig. 9.1 depicts the geographic locations of the networks in the above-mentioned experiments.

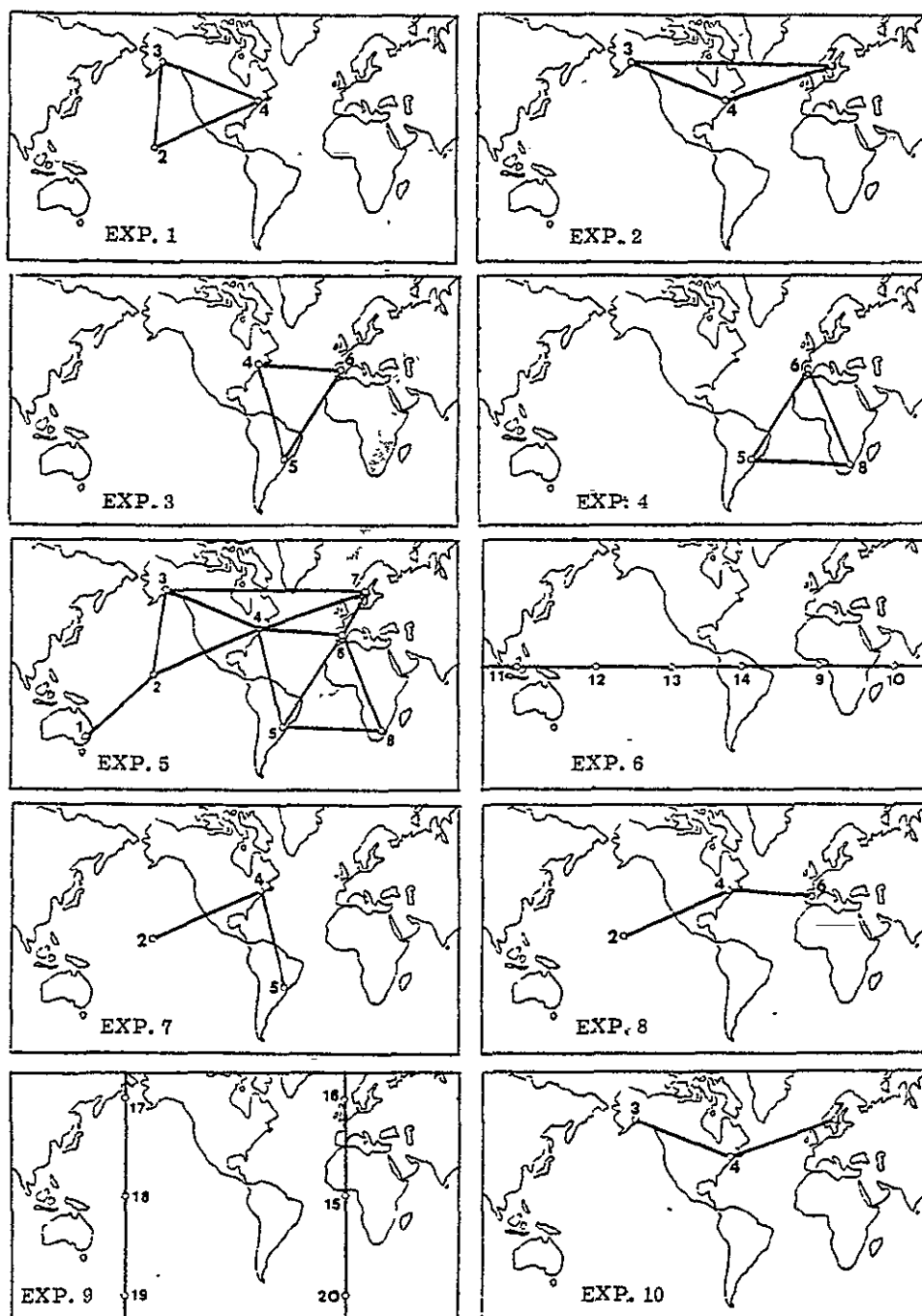


Fig. 9.1 Geographic locations of stations and baselines in Experiments 1 through 10.

9. Results and Conclusions

The results of the simulations are presented in three parts. The first part (Figs. 9.2 - 9.3) deals with the comparison of the ten designs described in the previous chapter, on the basis of one day of observations. Fig. 9.2 depicts the recovery (standard deviations) of earth rotation parameters ($\xi, \eta, \Xi, \Theta_0, \Omega$), while Fig. 9-3 shows the recovery of network geometry in terms of baseline lengths and angles (estimable quantities).

Standard deviations σ for each parameter are given as a function of the standard deviation σ_q of radio source coordinates and for various values of observational precision σ_b . Values used in the simulation are $\sigma_b, \sigma_q = 1, 5, 10, 20, 50$ cm. To achieve uniformity, σ_q is expressed as arc length on a reference sphere with a radius of 6371 km ($1'' \approx 30$ m).

The second part (Figs. 9.4 - 9.5) is a study of the improvement of the standard deviations of the parameters with the total time interval T of available observations. Station coordinates are assumed to be constant over T while earth rotation parameters are constant only over the subintervals of duration Δt (earth step). The ratio $\rho = \sigma/\sigma_b$ is given for each parameter as a function of T for various values of Δt . In the simulations, the following values have been used: $T = 1, 2, 3, 4, 5, 10, 30$ days; $\Delta t = 6, 12, 24$ hours. Experiment 2 has been chosen for this study. Radio source coordinate standard deviations are assumed to be $\sigma_q = \sigma_b$ for all cases. However, the variation of the results with σ_q is insignificant for $T \geq 5$ days.

The third part (Figs. 9.6 and 9.7) is a study of the variation of $\rho = \sigma/\sigma_b$ with Δt (earth step) based on one day of observations ($T = 1$ day) for some of the other designs (Experiments 1, 3, 4, 6, 9, 10) is shown.

Since the results of the simulations are fully presented here, one may draw his own conclusions. However, the following general remarks are made in an attempt to summarize the results:

(1) Earth Rotation Parameters from One-Day Observations

For one day of observations and the recovery of earth rotation parameters, the following are noted:

(a) Three-baseline "triangle" designs (Experiments 1, 2, 3, 4 in Fig. 9.2) are capable of recovering polar motion (ξ, η) with a precision of the same order of magnitude as that of the observations, even for low radio source accuracies ($\sigma_q = 50$ cm).

For two-baseline designs (Experiments 7, 8, 10 in Fig. 9.2), polar motion recovery is of the same quality as for the "triangle" station configurations. However, the removal of one baseline in Experiment 10 results in an increase of polar motion standard deviations by a factor of 1.5 compared to Experiment 2. The more complex design of Experiment 5 offers no dramatic improvement in polar motion recovery. An essential improvement can be seen in the equatorial design of Experiment 6 and even more in Experiment 9 which is the "best," but admittedly not very practical design for polar motion recovery.

(b) The standard deviations of the (precession-nutation) parameters Ξ and H are practically the same for all two- or three-baseline designs (Experiments 1, 2, 3, 4, 7, 8, 10). In particular, the removal of one baseline in Experiment 10 with respect to Experiment 2 has no effect on the recovery of Ξ and H . The best recovery of Ξ , H can be seen in Experiments 5 and 9. Although the station configuration in Experiment 6 is of the same shape as in Experiment 9, the change in their position (equatorial vs. meridian) has a great effect on Ξ , H standard deviations. Experiment 6 has the weakest recovery, while Experiment 9 provides the strongest one.

(c) The standard deviation of Θ_0 varies with the station configurations. It is highest for Experiments 4, 8, 7; medium for Experiments 10, 3, 5; and smallest for Experiments 1, 2, 9, and especially 6 (best).

(d) Recovery of Ω varies strongly even for designs of the same type. It is generally weak for two-baseline designs (Experiments 7, 8, 10) and somewhat better for three-baseline designs with the exception of the extremely weak recovery in Experiment 4. Many-station designs (Experiments 5, 6, 9) improve the recovery especially in the equatorial configuration of Experiment 6.

(e) Ω and Θ_0 are the only parameters favored by the artificial equatorial design of Experiment 6 vs. the meridian one of Experiment 9. Recovery of the two components ξ and η of polar motion is not equal but depends on the position of the stations in each experiment with the exception of Experiment 6. On the contrary, both the Ξ and H parameters of precession-nutation are recovered with practically the same precision for every design.

The recovery of earth rotation parameters varies with the accuracy (σ_0) of radio source coordinates. However, this dependence is much stronger for Ξ , H ; milder for the rest of the parameters; and even disappearing for Ω , ξ , η in the strongest designs of Experiments 6 and 9.

(f) The results of the idealized designs of Experiments 6 and 9 can be used to verify some of the results of Chapter 6. Recovery of ξ , η , Ξ , H is better for Experiment 9 than 6, while recovery of Θ_0 , Ω is better for Experiment 6. This is explained by the fact that Experiment 9 involves baselines parallel to the equator (16-17, 19-20) observing polar radio sources, a design sensitive to ξ , η , Ξ , H . Experiment 6 involves equatorial baselines observing equatorial radio sources, the design most sensitive to Θ_0 , Ω (see Table 6.1).

(g) The recovery of baselines varies with station configuration. However, larger number of baselines does not in general improve the individual recovery as in the case of earth rotation parameters. For example, recovery in the three-baseline design of Experiment 2 is much better than in any of the many-station designs (Experiments 5, 6, 9). The relative recovery of angles in the same experiment is inversely proportional to the recovery of the opposite baselines. Dependence on radio source accuracy varies and disappears in the case of the equatorial configuration of Experiment 6, especially in comparison to the meridian configuration of Experiment 9.

(2) Earth Rotation and Baseline Parameters As a Function of the Length of Observations and Earth Step

For the variation of the recovery of earth rotation and baseline parameters with respect to the total time interval of observations T , it is noted that the standard deviations σ of the parameters are directly proportional to the standard deviations σ_b of the observations. (Note that $\sigma_0 = \sigma_b$.) For this reason only the ratio $\rho = \sigma/\sigma_b$ is depicted in Figs. 9.4 and 9.5.

The dependence of ρ on the earth step Δt is very strong for earth rotation parameters, especially for Ω (see Fig. 9.4), while not significant for baseline

parameters (see Fig. 9.5). In general, the recovery improves as Δt increases since the corresponding number of unknown parameters in the adjustment decreases. For earth rotation parameters (Fig. 9.4), standard deviation decreases strongly as T increases from 1 to 5 days. An additional increase of T from 5 to 10 days offers a small improvement, while no significant additional improvement is gained from the extension of T from 10 to 30 days. The same is true for baseline parameters (lengths and angles, see Fig. 9.5) with the exception of an additional small improvement with the increase of T from 10 to 30 days.

(3) Earth Rotation and Baseline Parameters As a Function of Earth Step

As mentioned, the recovery of earth rotation and network geometry parameters improves with larger earth step Δt (Figs. 9.6 and 9.7). The only exceptions are with respect to the ξ, η parameters in Experiments 1, 2, 4, 10 and with respect to Θ_0 in Experiment 4 where $\Delta t = 24$ hours results in worse recovery than $\Delta t = 12$ hours or even $\Delta t = 6$ hours in some cases. This "inversion" is correlated to the weak recovery of polar motion in Experiments 1, 2, 4, 10 (see Fig. 9.2) and to the weak recovery of Θ_0 in Experiment 4. On the contrary, for Experiments 6 and 9, where ξ, η and Θ_0 recovery is better, variation of ρ with respect to Δt is "normal." Such variation is negligible for Θ_0 in Experiment 6 and also for network geometry parameters in Experiments 6 and 9. Naturally an increase in the length of the earth step is a disadvantage when short periodic variations in the earth rotation parameters are sought.

Fig. 9.2 Recovery of earth rotation parameters, Experiments 1 - 10.

Total time span of observations: $T = 1$ day

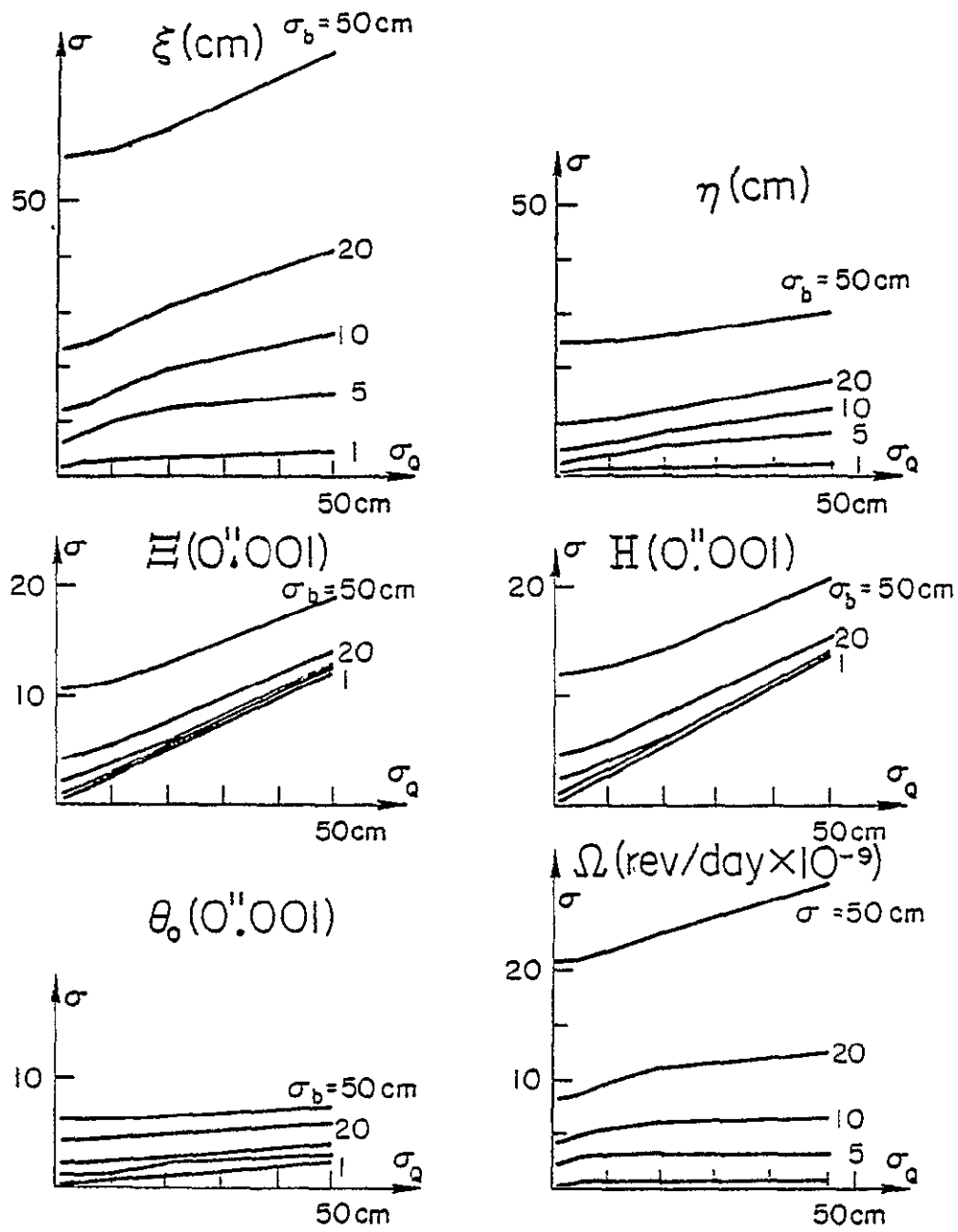
Earth step: $\Delta t = 1$ day

σ = a posteriori standard deviation of earth rotation parameters

σ_q = a priori standard deviation of radio source coordinates

σ_b = standard deviation of observational errors

Note: Angular quantities when expressed in length units are
arc lengths on a sphere of radius $R = 6\,371\,000$ m
($1'' \approx 30$ m).



EXPERIMENT 1

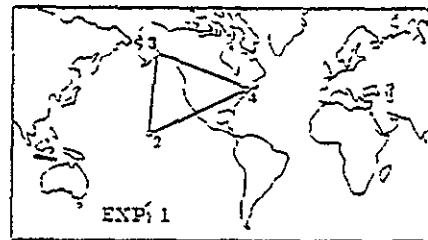
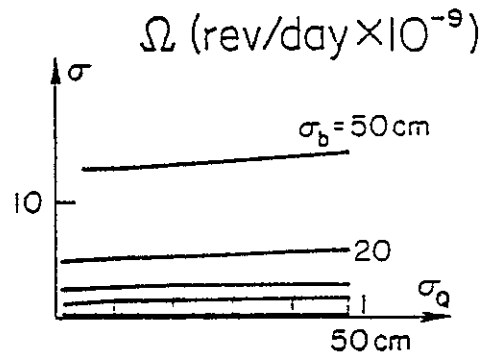
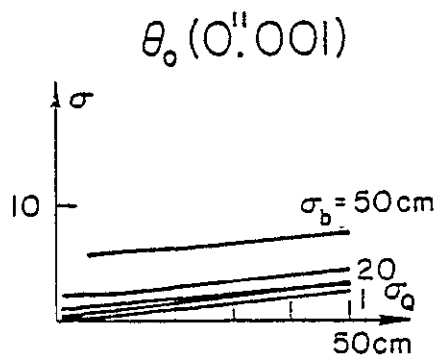
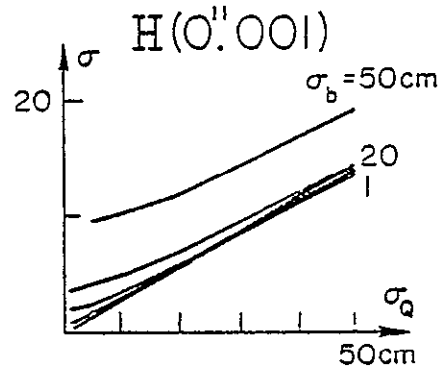
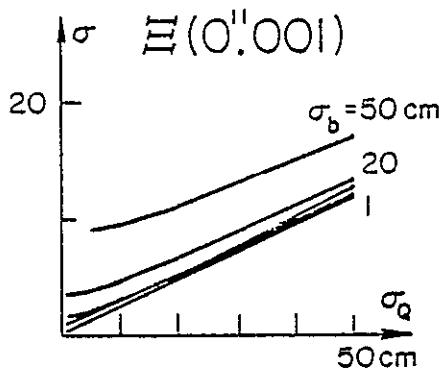
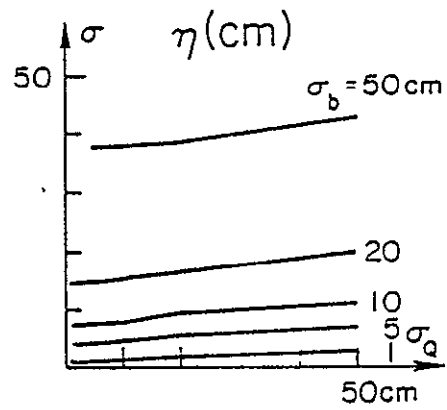
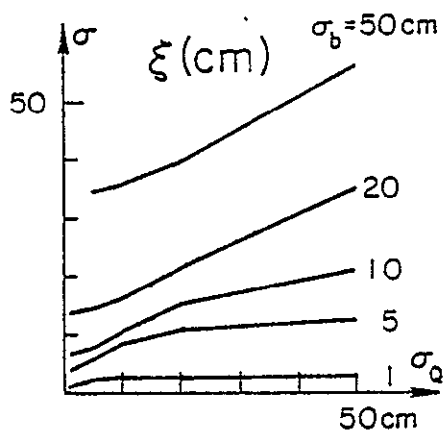


Fig. 9.2 Recovery of earth rotation parameters.



EXPERIMENT 2

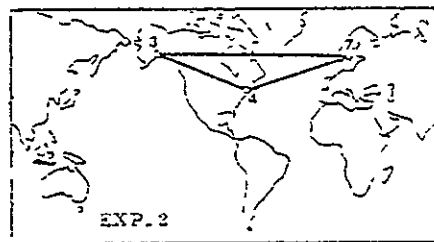
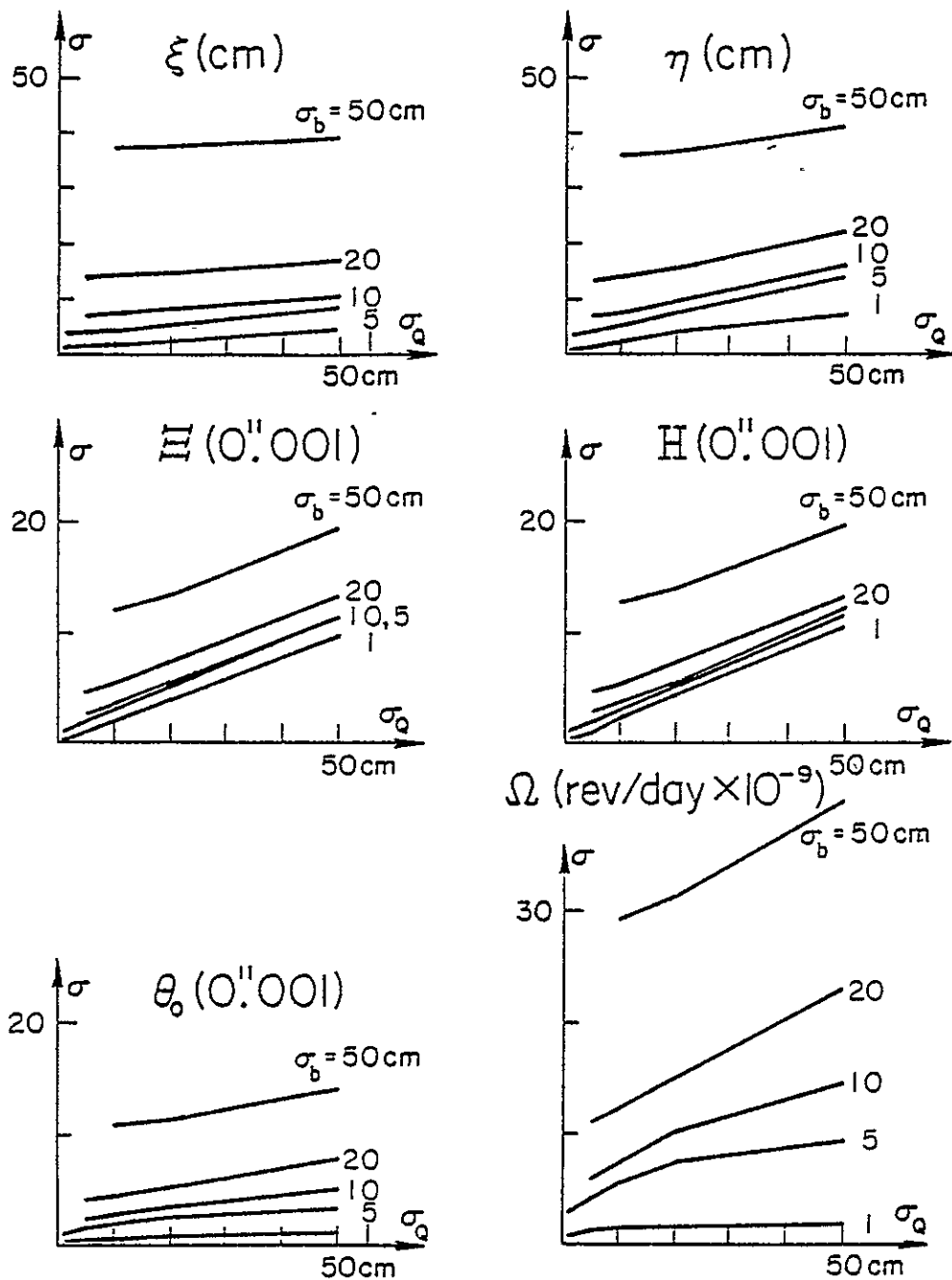


Fig. 9.2 (cont'd)



EXPERIMENT 3

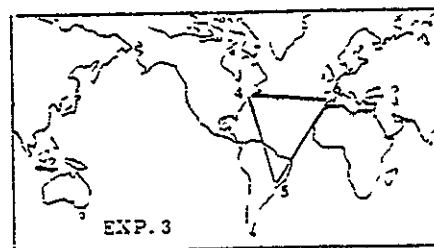
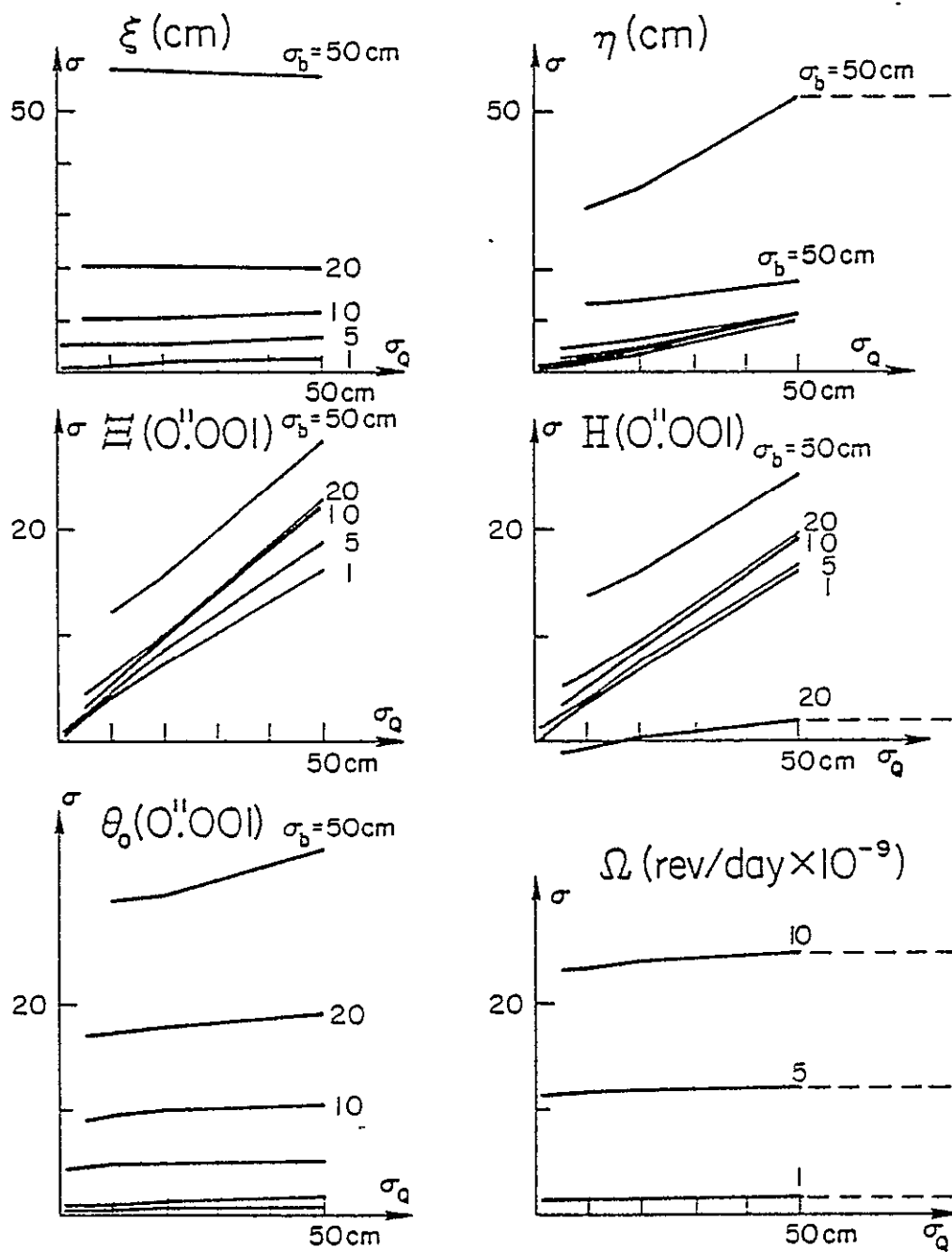


Fig. 9.2 (cont'd)



EXPERIMENT 4

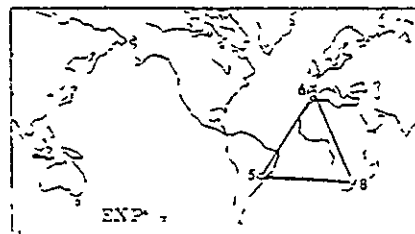
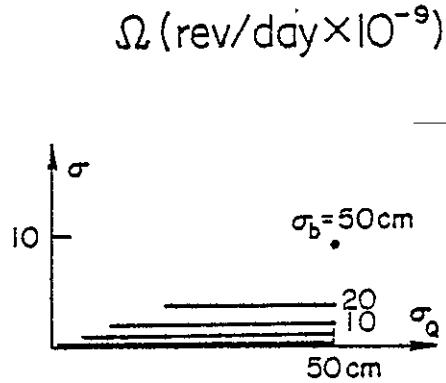
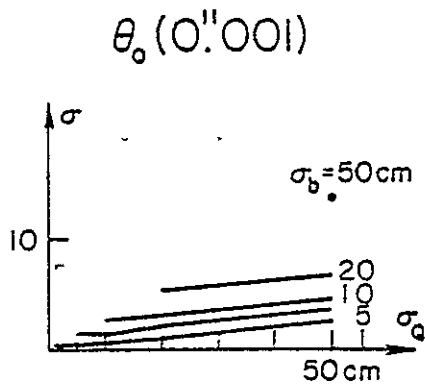
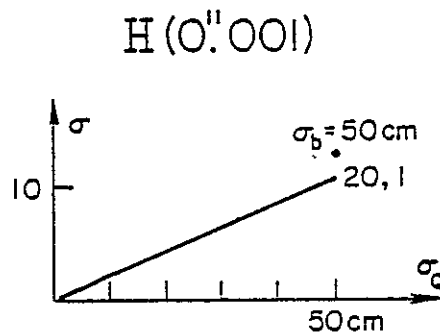
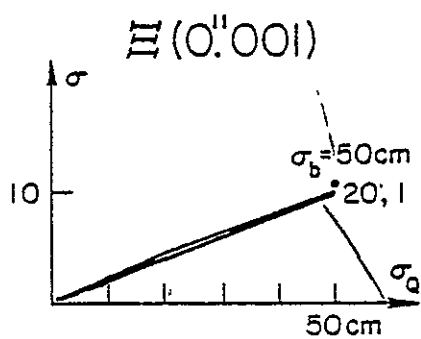
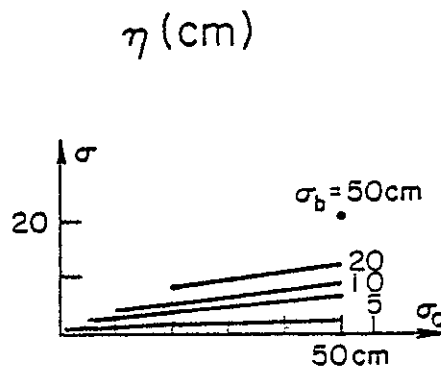
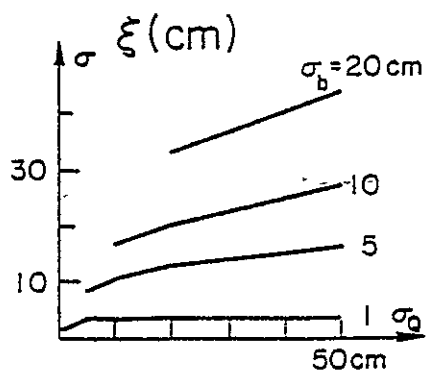


Fig. 9.2 (cont'd)



EXPERIMENT 5

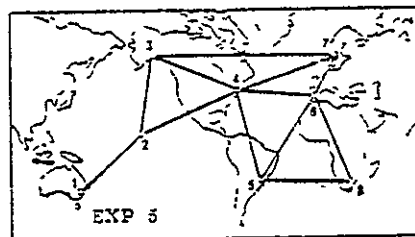
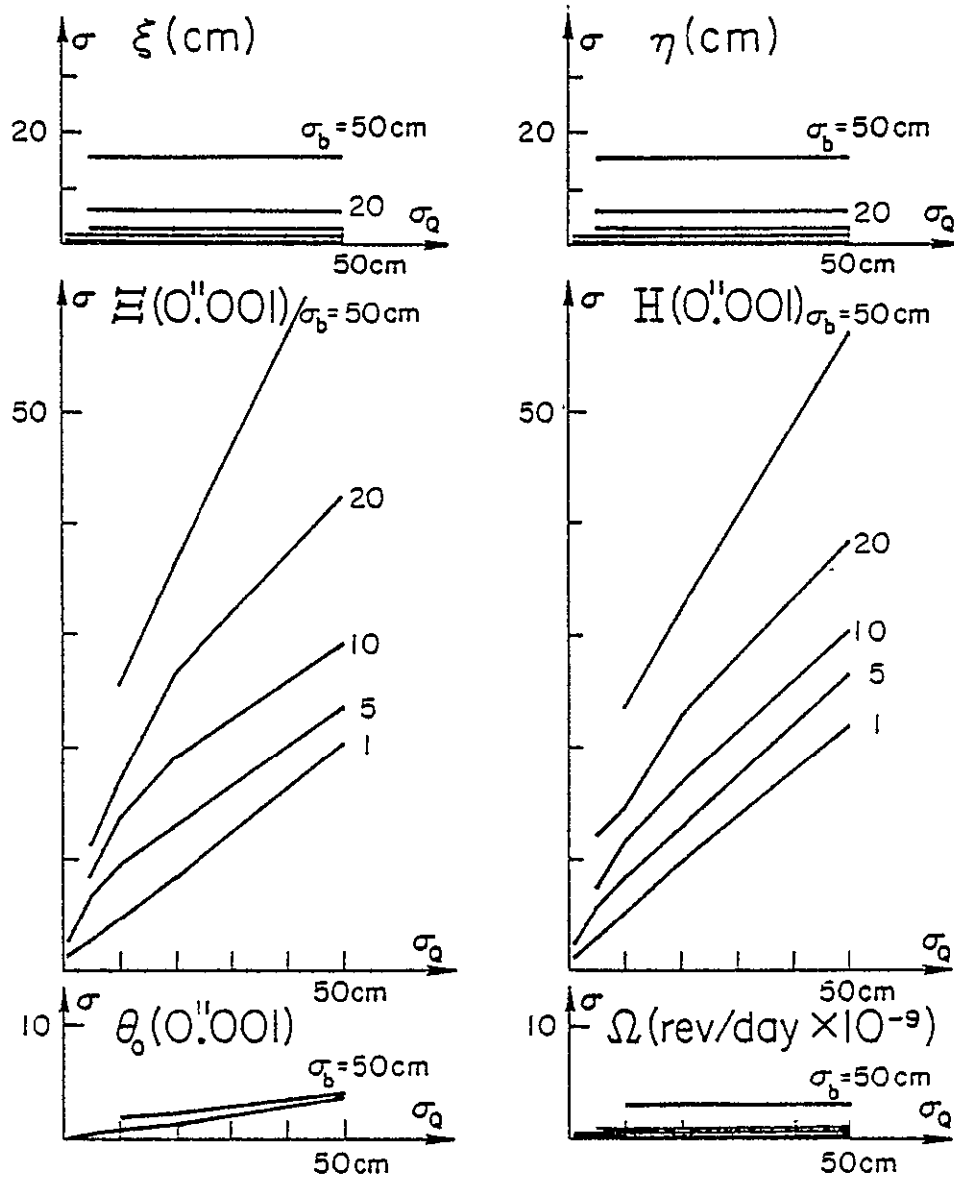


Fig. 9.2 (cont'd)



EXPERIMENT 6

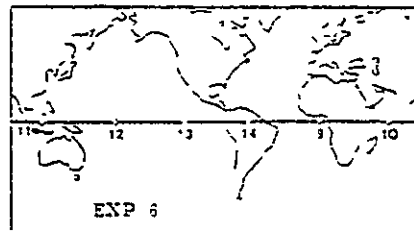
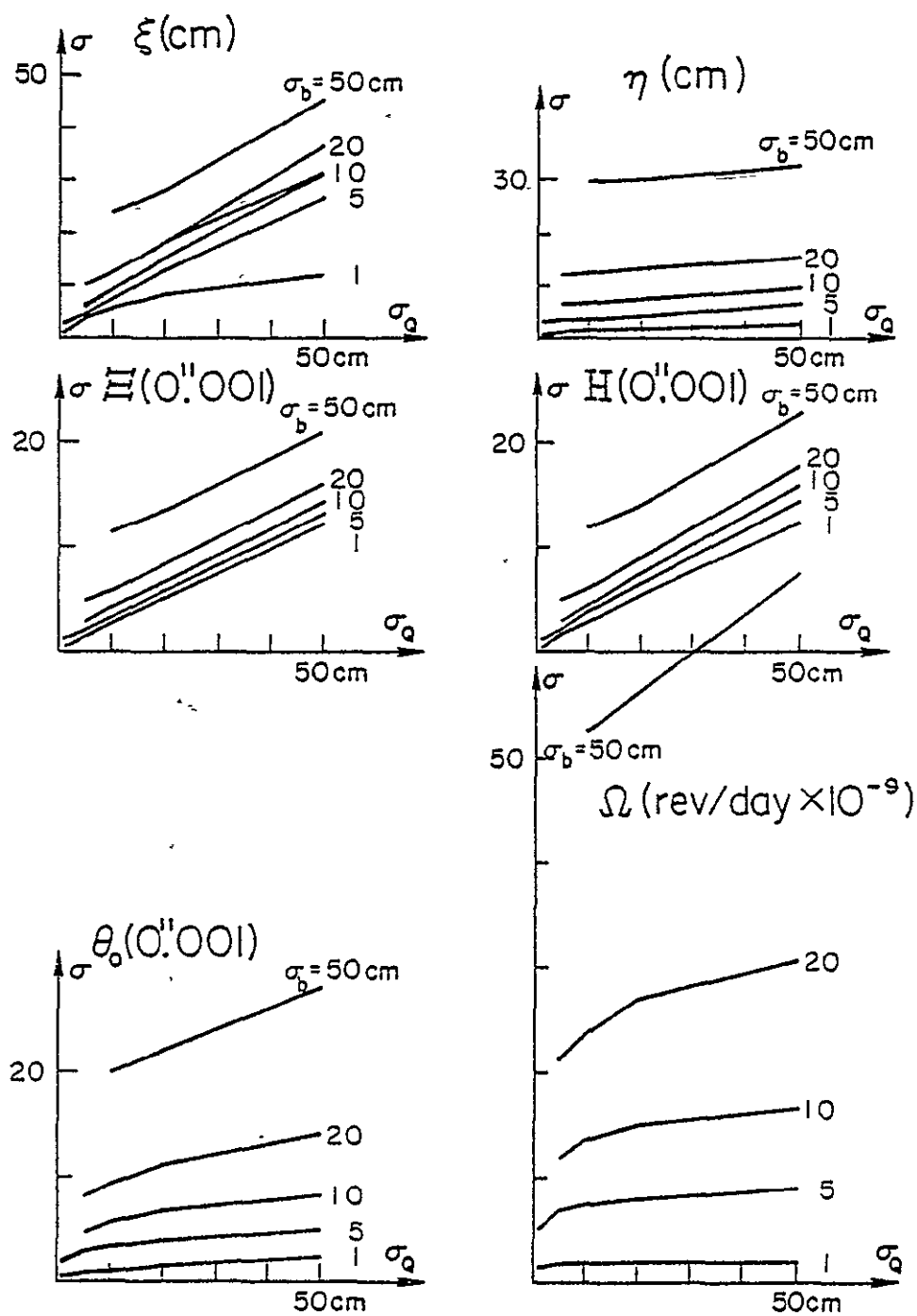


Fig. 9.2 (cont'd)



EXPERIMENT 7

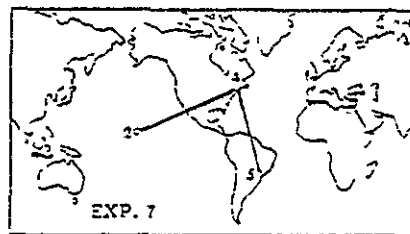
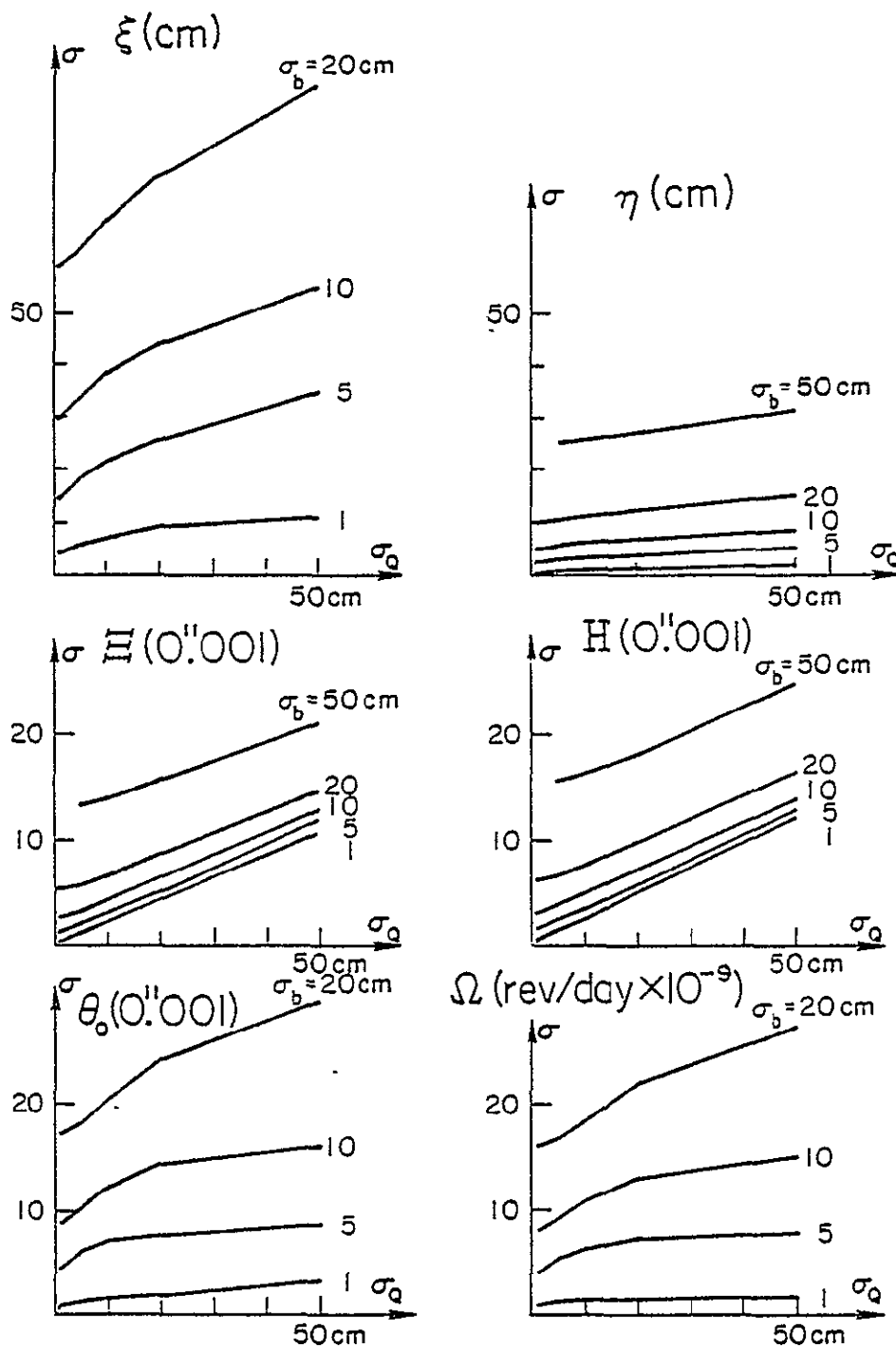


Fig. 9.2 (cont'd)



EXPERIMENT 8

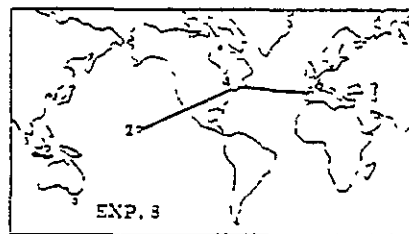
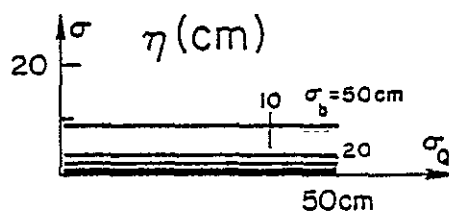
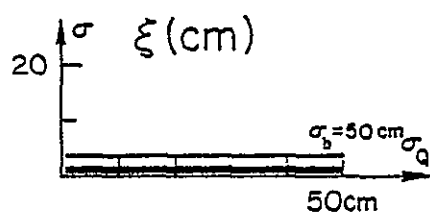
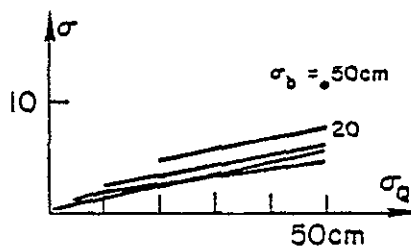
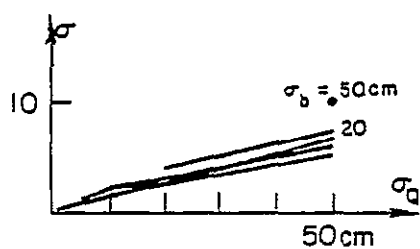


Fig. 9.2 (cont'd)



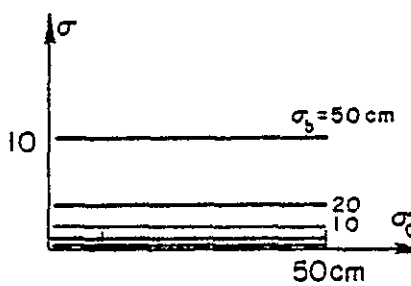
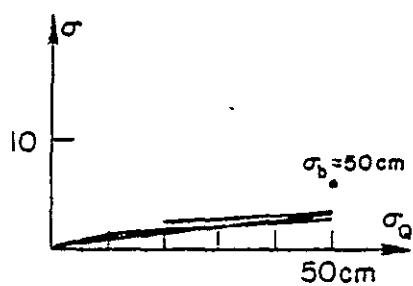
$\Xi(0''.001)$

$H(0''.001)$



$\theta_0(0''.001)$

$\Omega \text{ (rev/day} \times 10^{-9}\text{)}$



EXPERIMENT 9

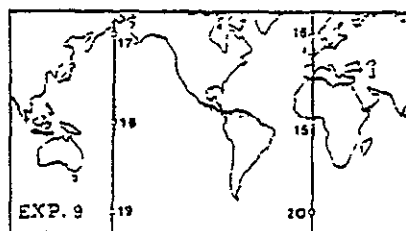
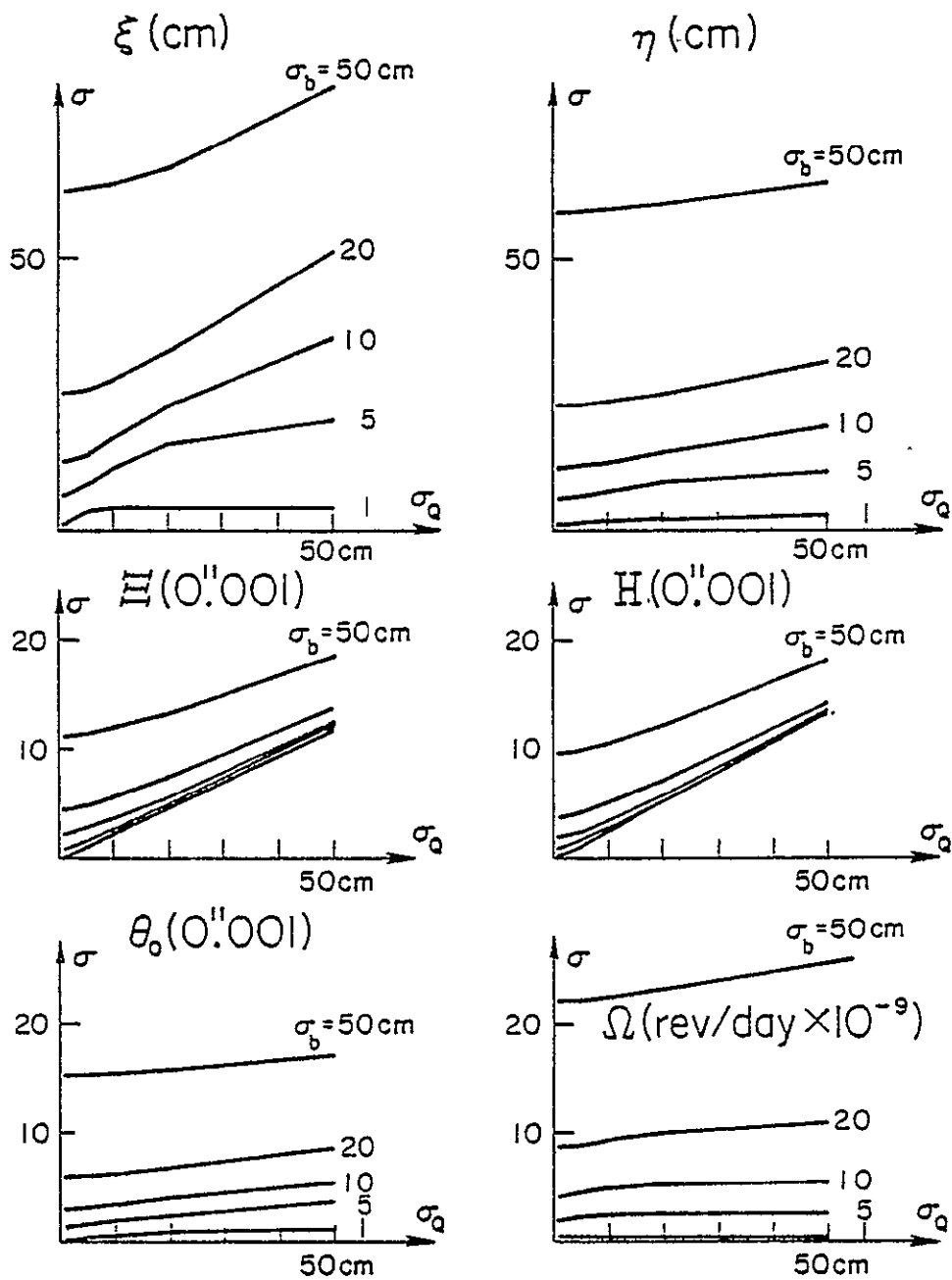


Fig. 9.2 (cont'd)



EXPERIMENT 10

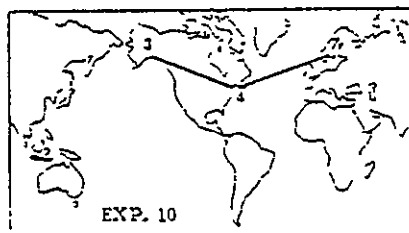


Fig. 9.2 (cont'd)

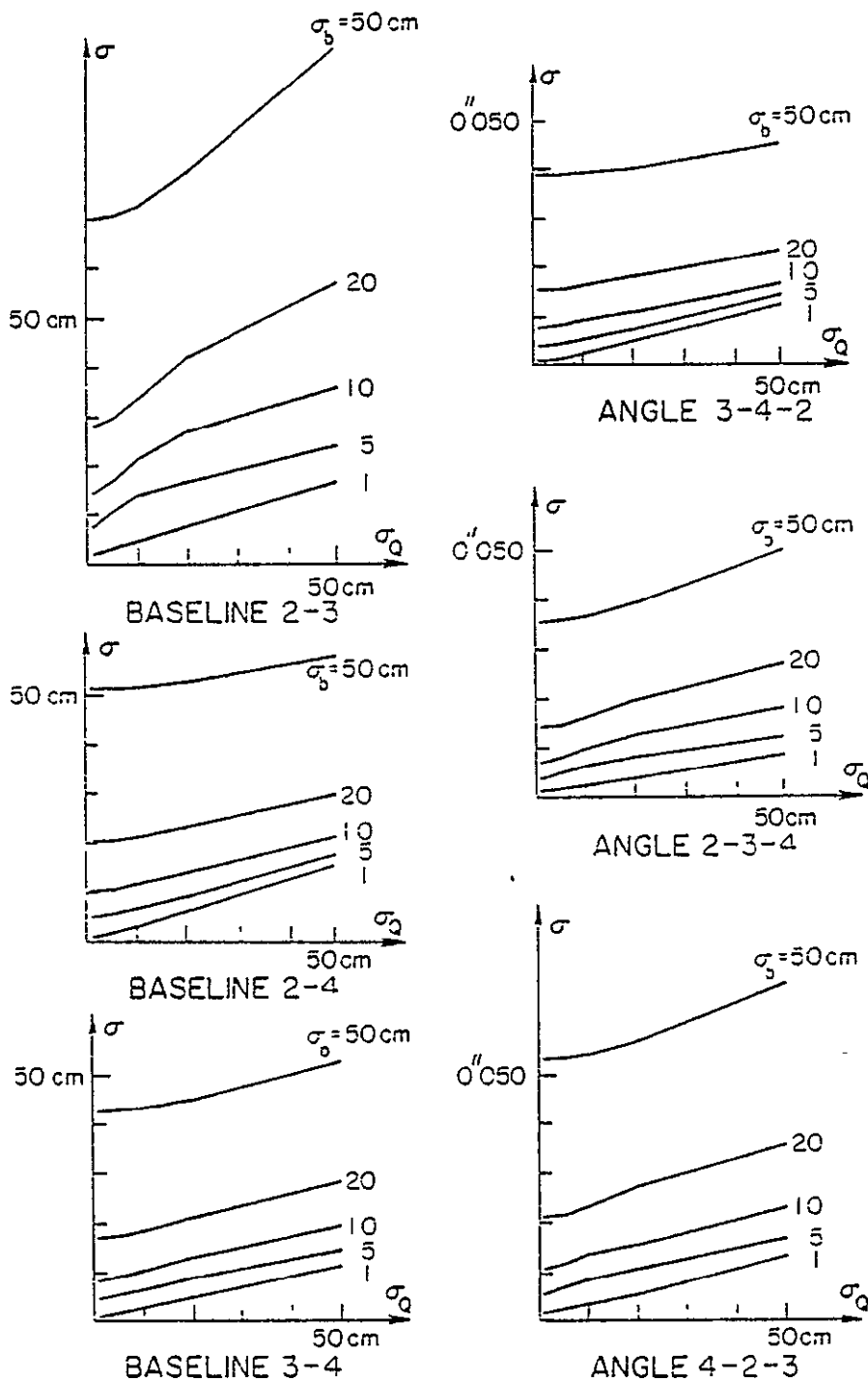
Fig. 9.3 Recovery of baseline lengths and angles, Experiments 1 - 10

Total time span of observations: $T = 1$ day

Earth step: $\Delta t = 1$ day

For explanation of other symbols, see Fig. 9.2, p. 40.

ORIGINAL PAGE IS
OF POOR QUALITY



EXPERIMENT 1

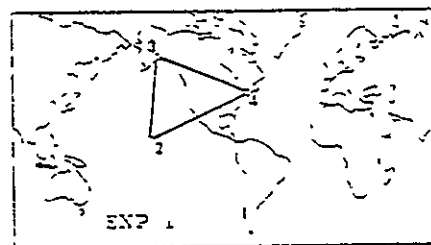
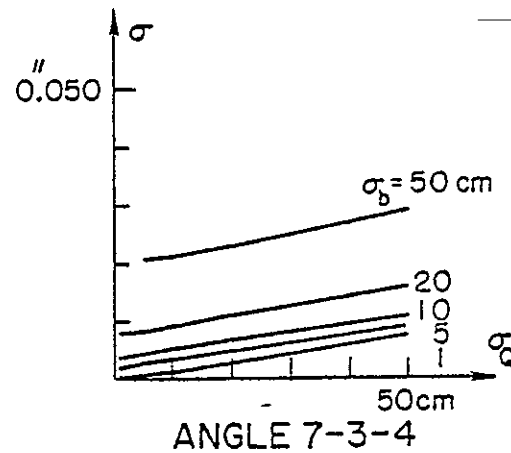
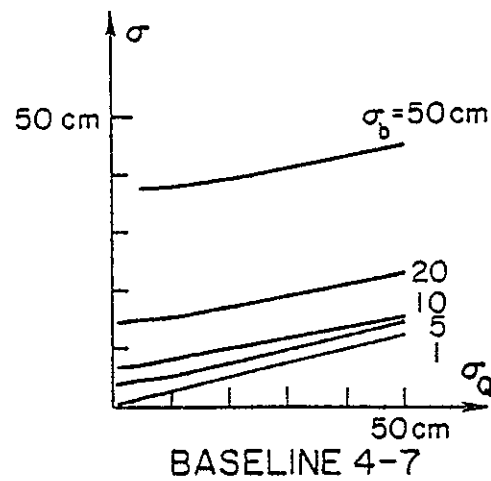
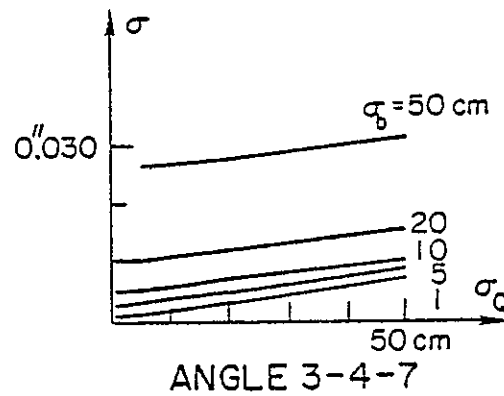
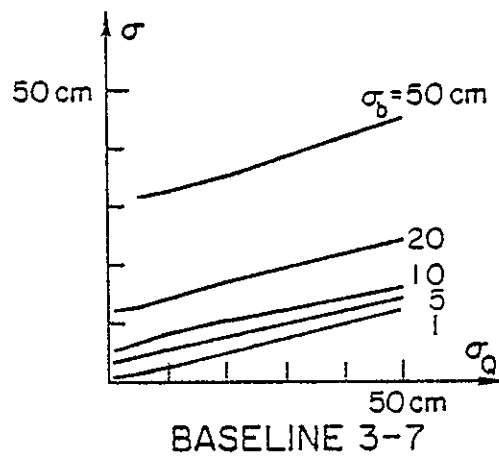
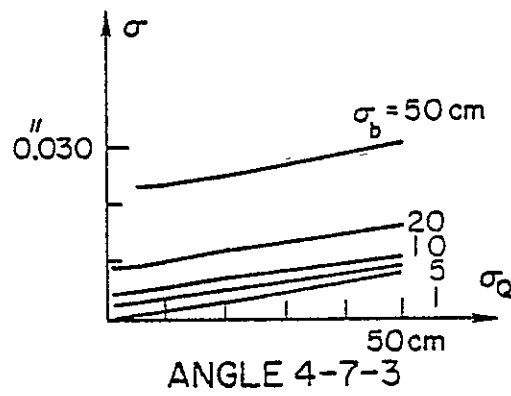
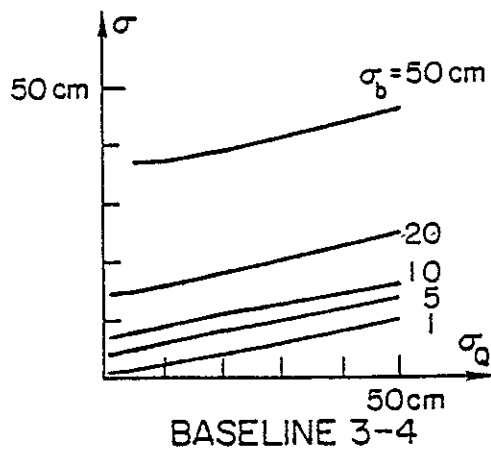


Fig. 9.3 Recovery of baseline lengths and angles.



EXPERIMENT 2

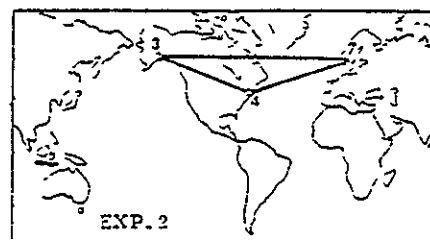
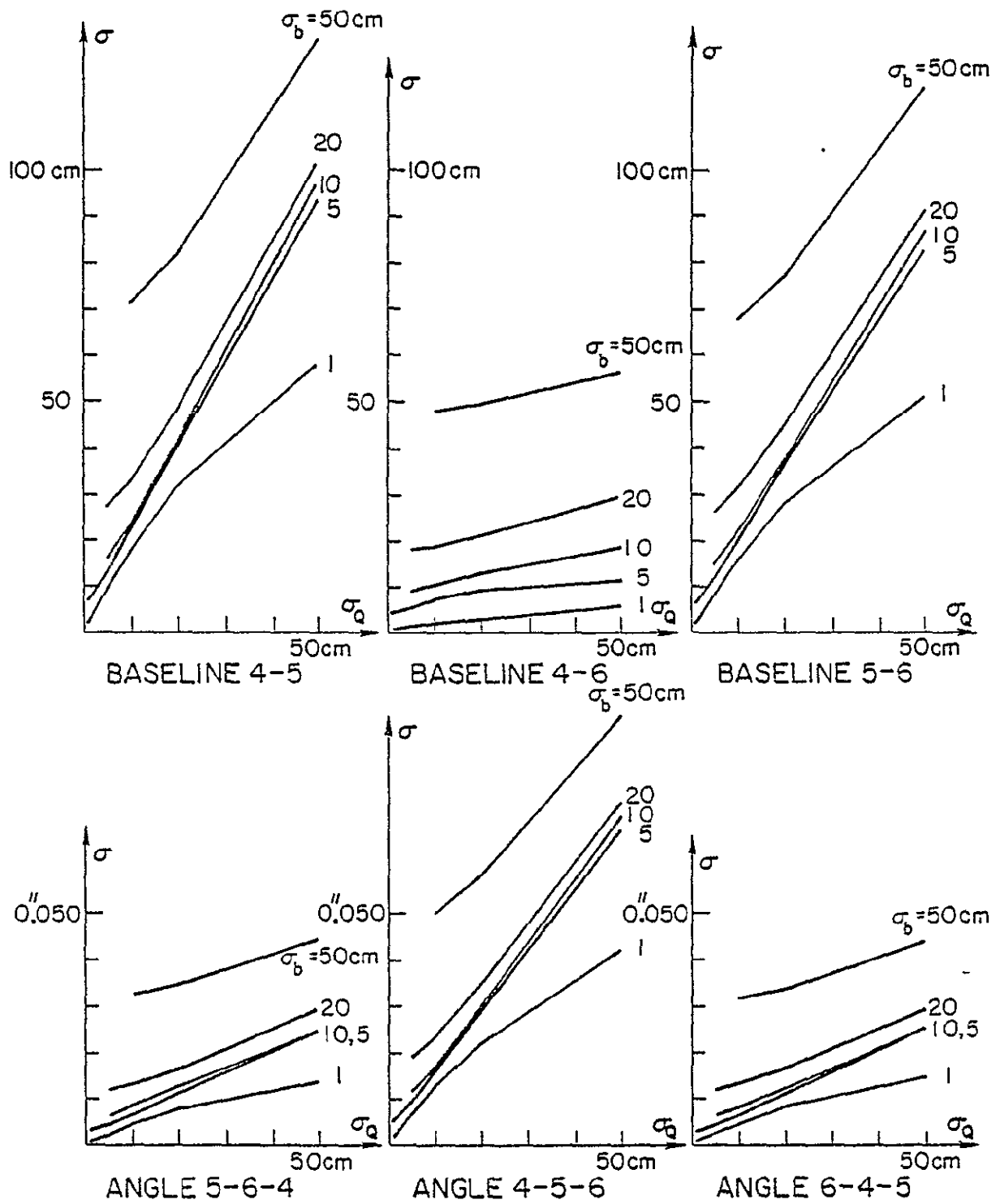


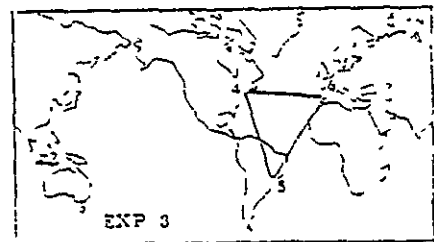
Fig. 9.3 (cont'd)

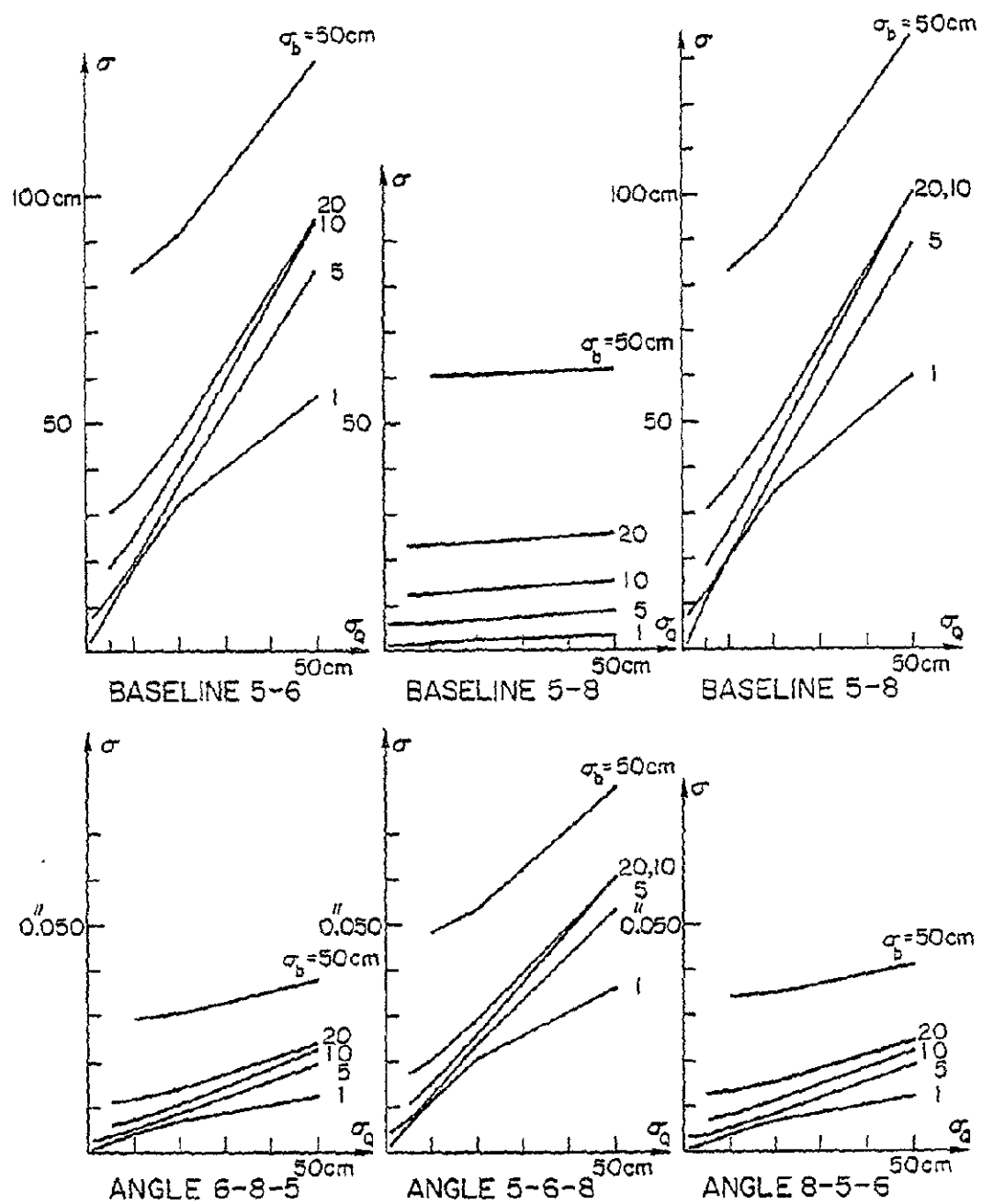


ORIGINAL PAGE IS
OF POOR QUALITY

EXPERIMENT 3

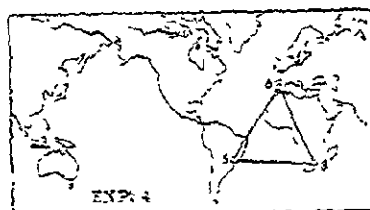
Fig. 9.3 (cont'd)

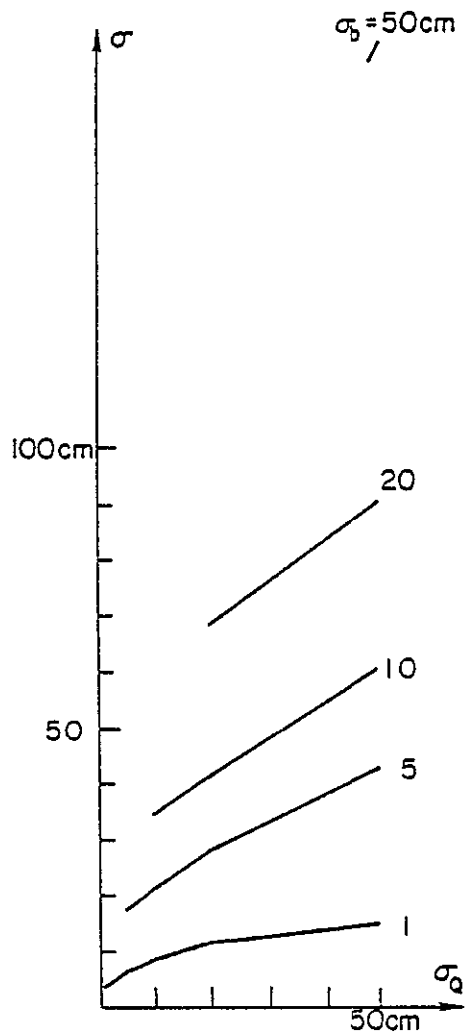




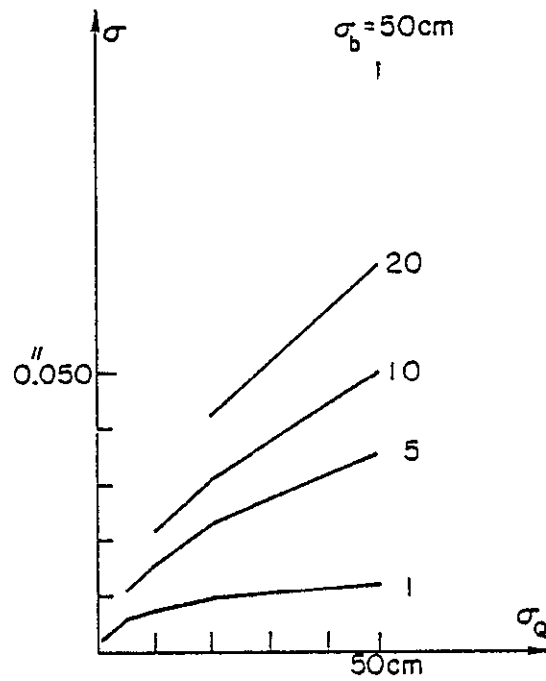
EXPERIMENT 4

Fig. 9.3 (cont'd)

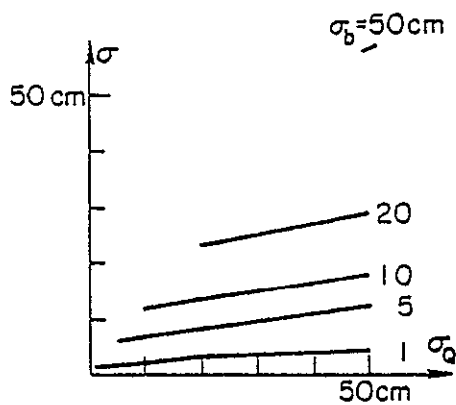




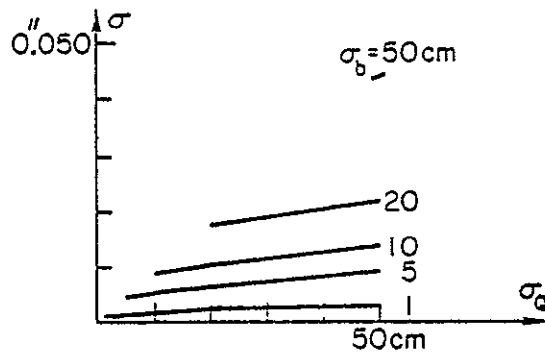
BASELINE 1-2 (WORST)



ANGLE 6-8-4 (WORST)



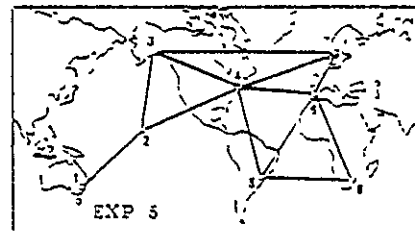
BASELINE 4-7 (BEST)



ANGLE 3-4-7 (BEST)

EXPERIMENT 5

Fig. 9.3 (cont'd)



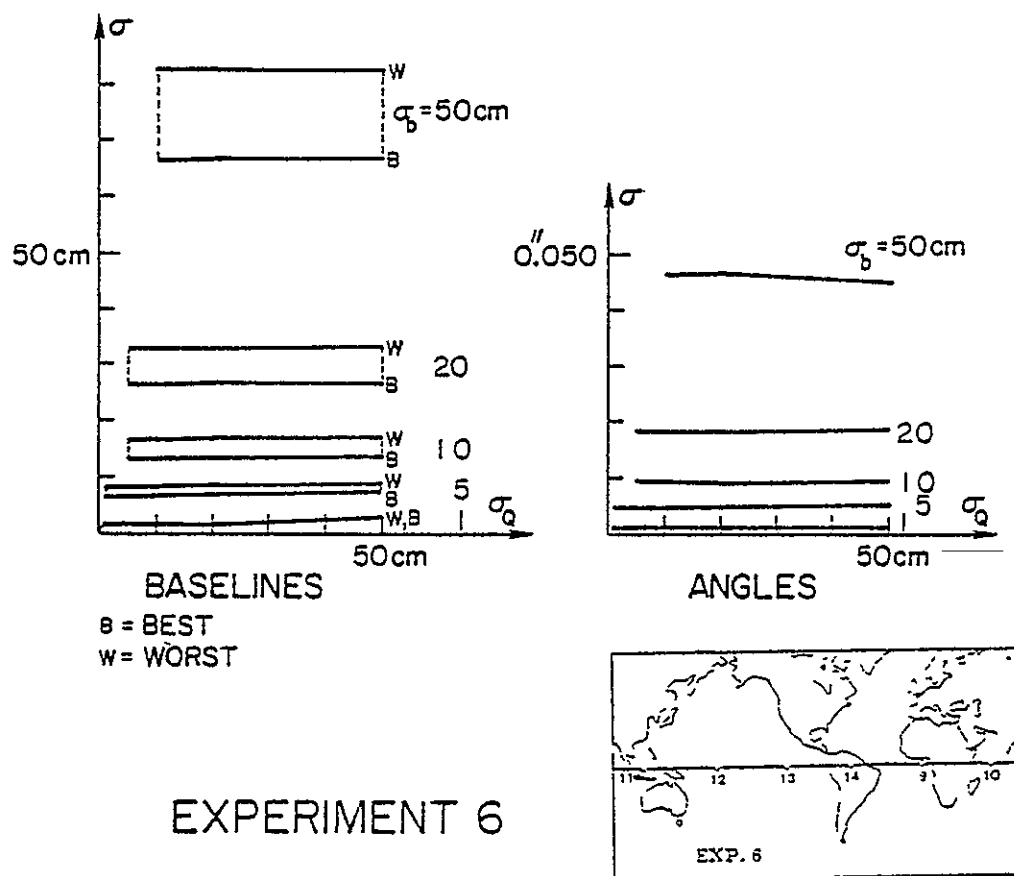
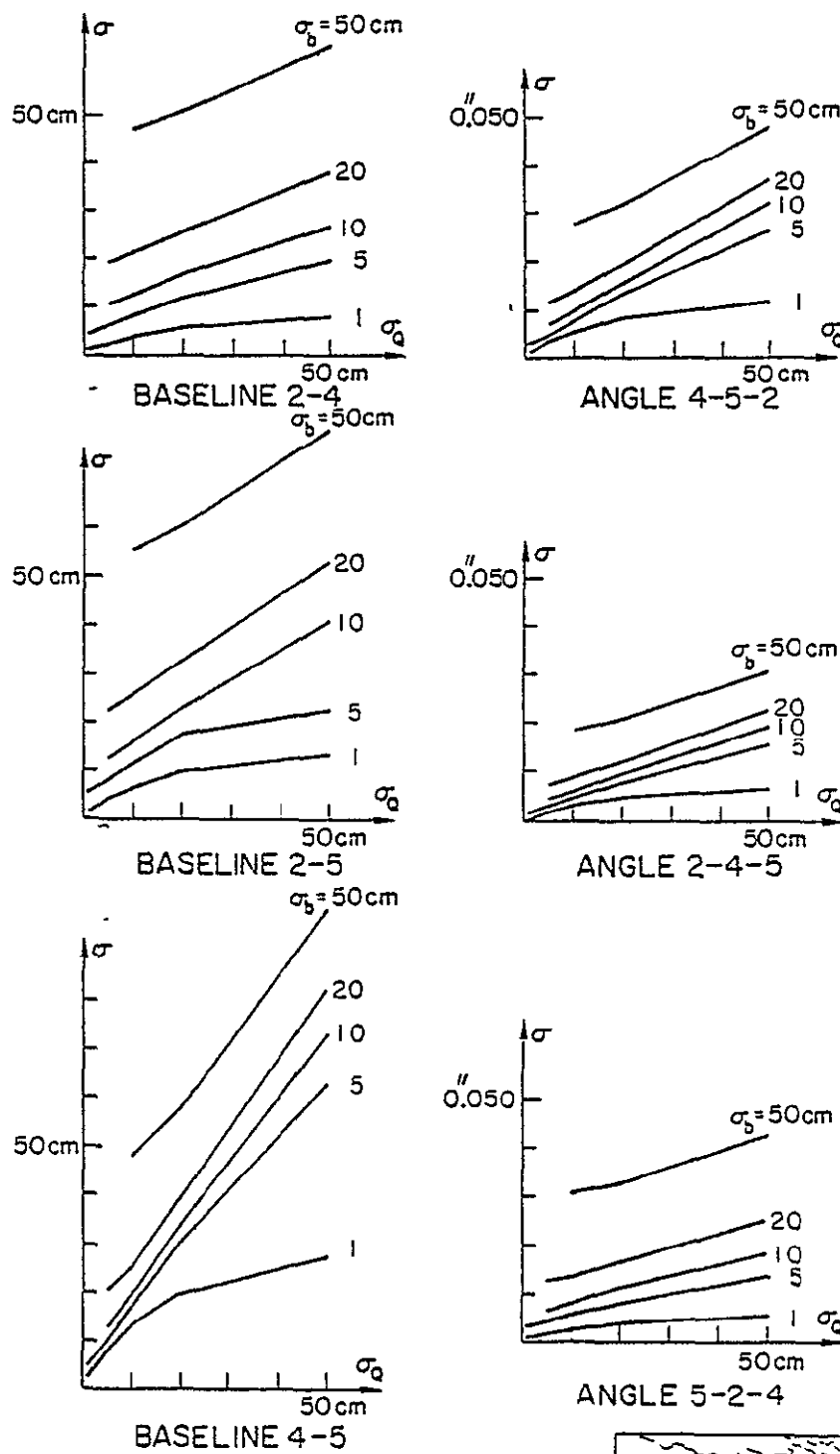
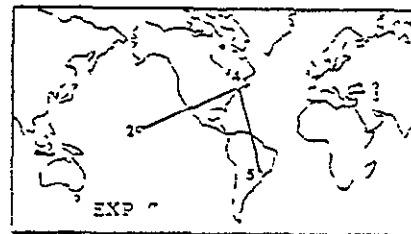


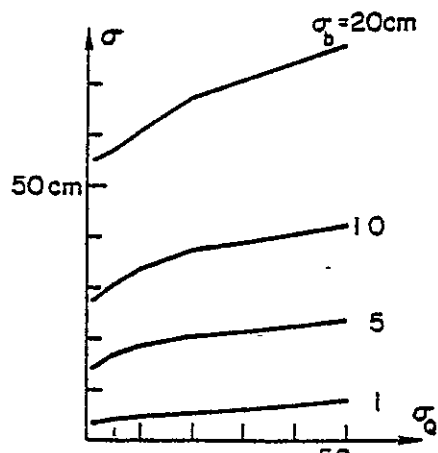
Fig. 9.3 (cont'd)



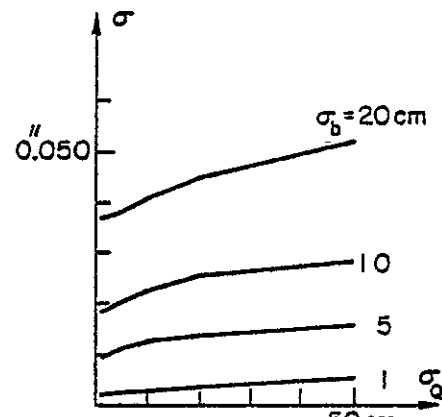
EXPERIMENT 7

Fig. 9.3 (cont'd)

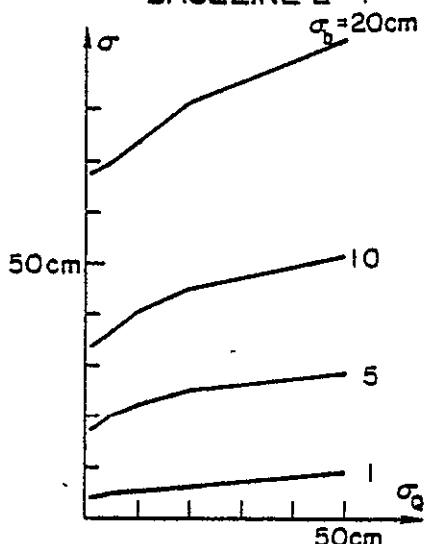




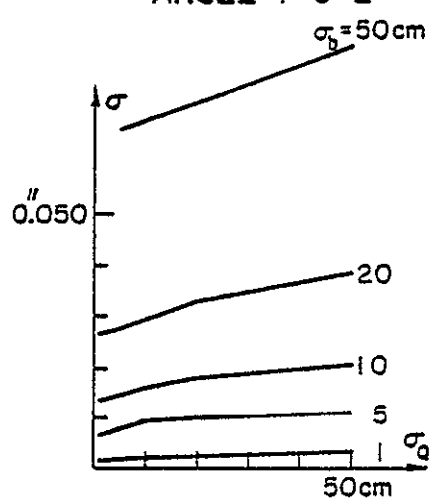
BASELINE 2-4



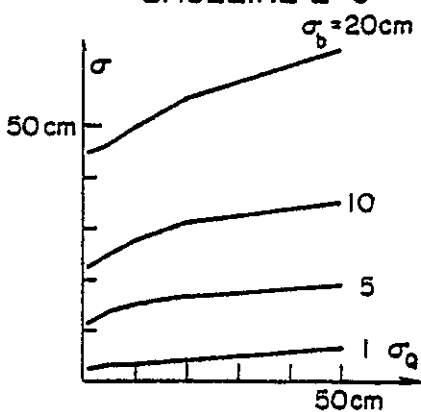
ANGLE 4-6-2



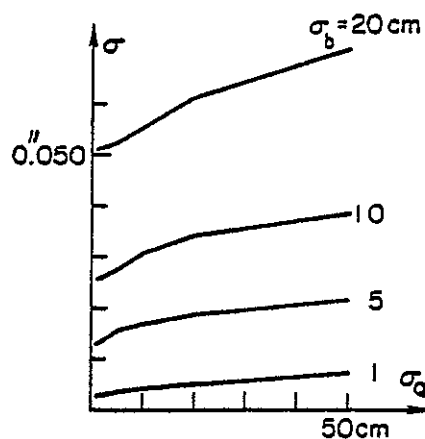
BASELINE 2-6



ANGLE 2-4-6



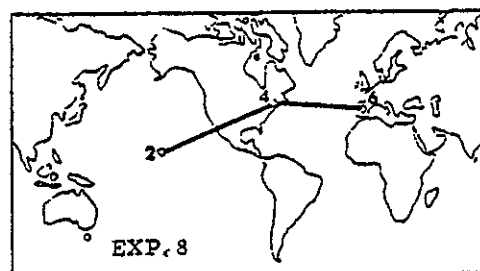
BASELINE 4-6

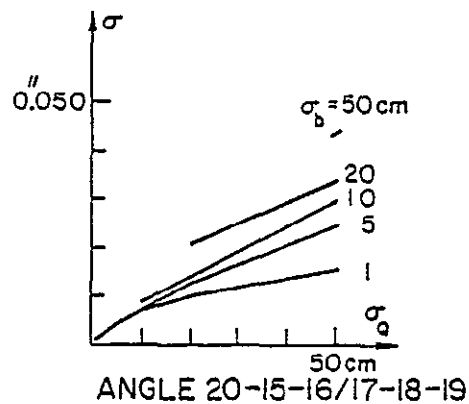
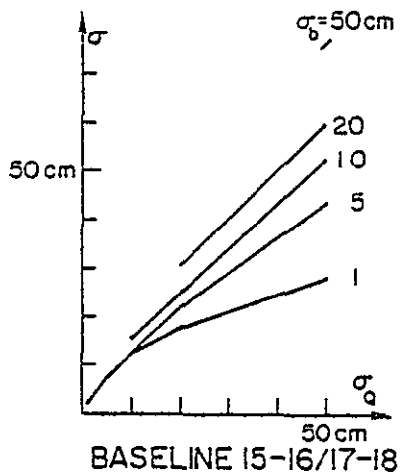
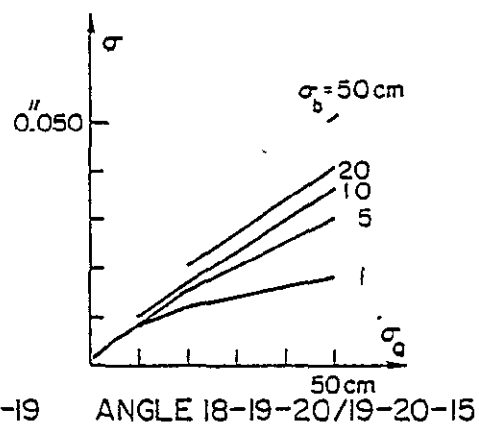
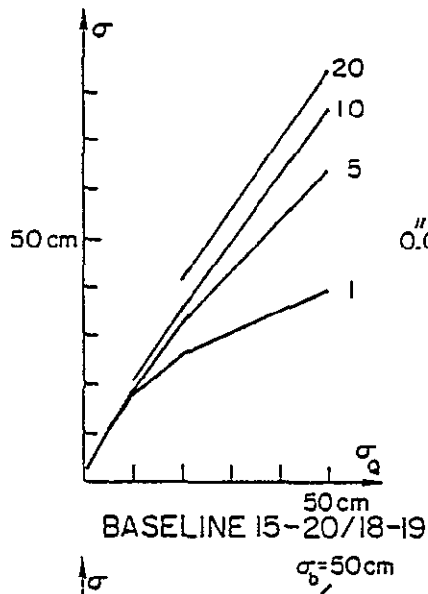
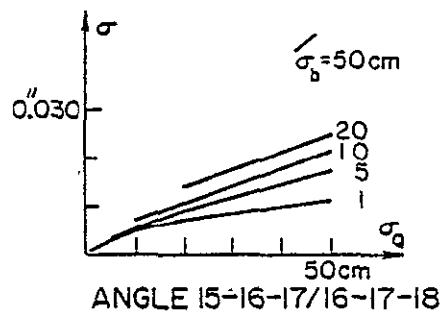
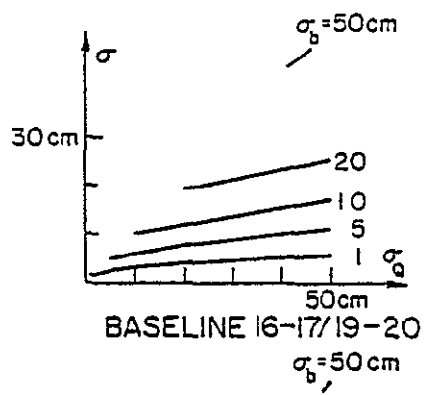


ANGLE 6-2-4

EXPERIMENT 8

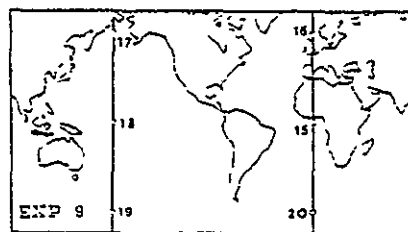
Fig. 9.3 (cont'd)

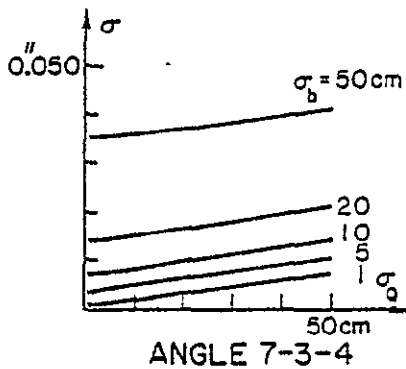
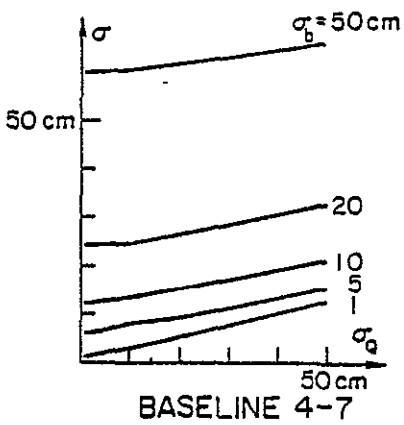
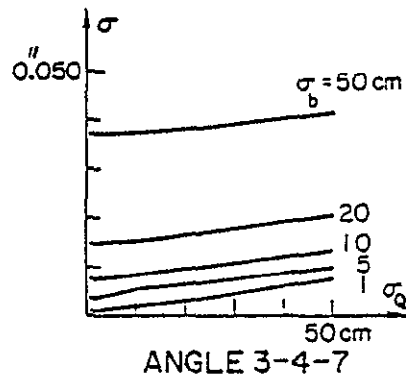
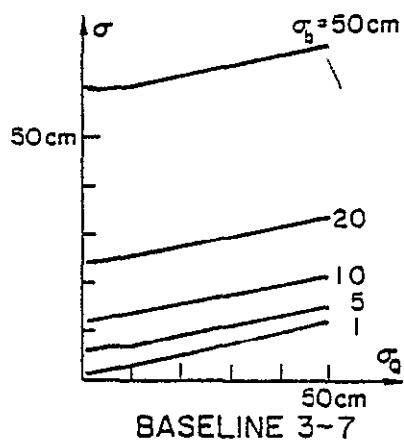
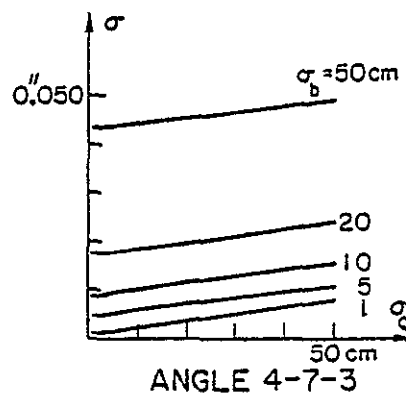
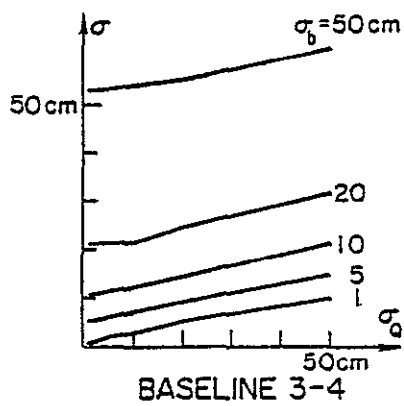




EXPERIMENT 9

Fig. 9.3 (cont'd)





EXPERIMENT 10

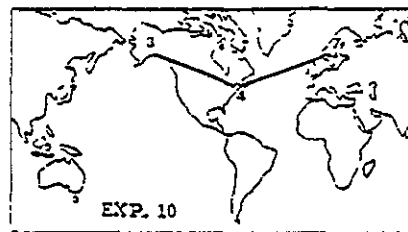


Fig. 9.3 (cont'd)

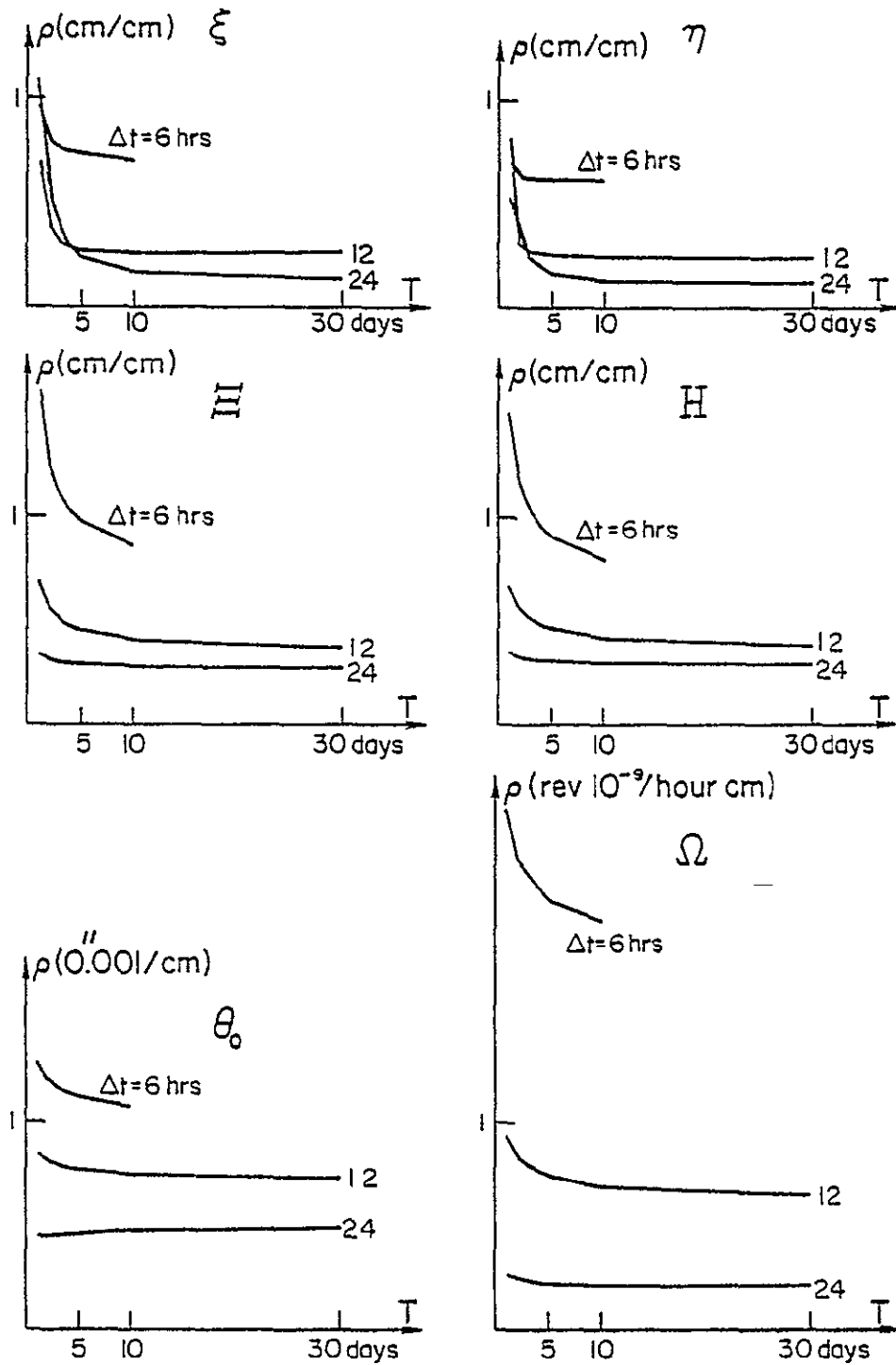


Fig. 9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2).

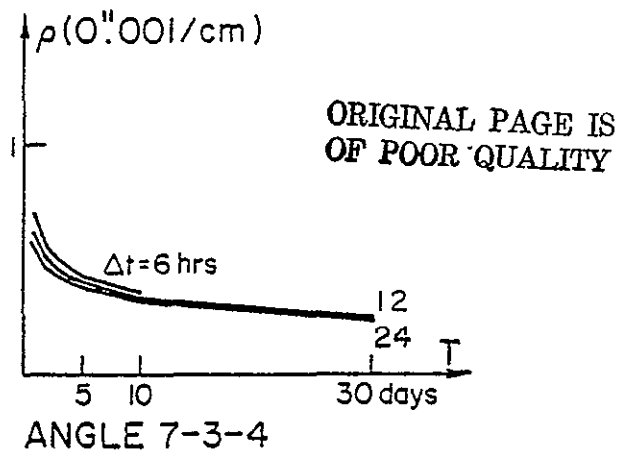
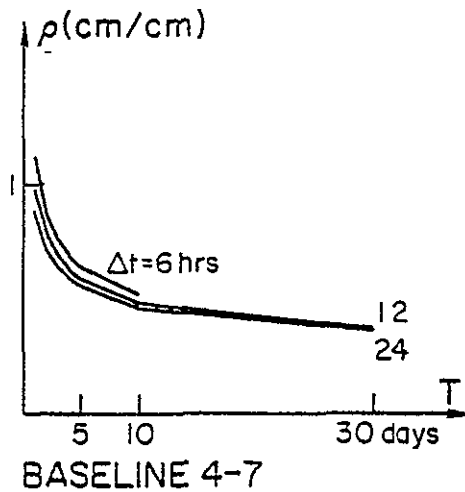
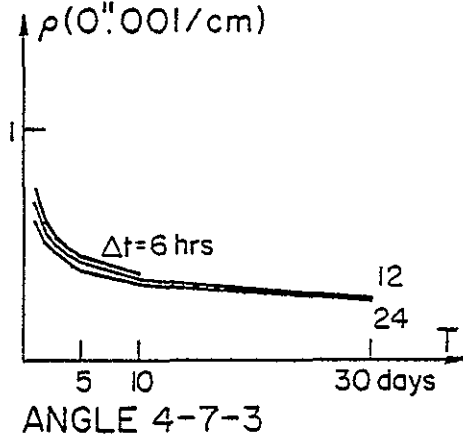
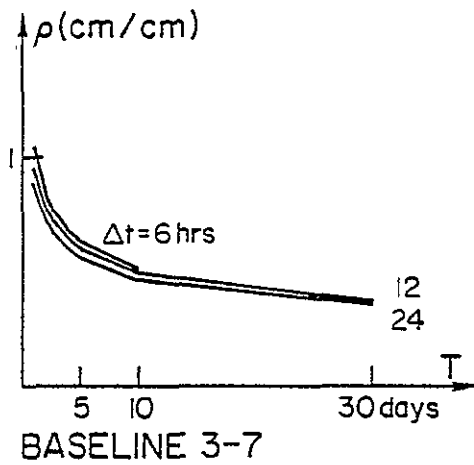
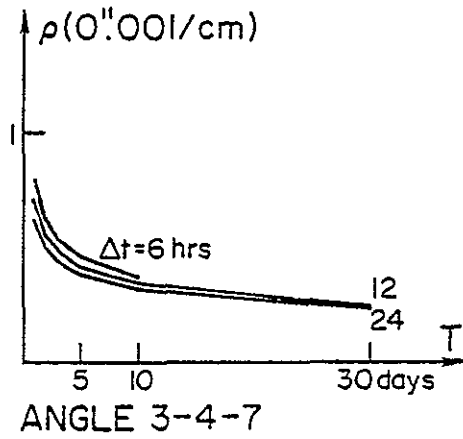
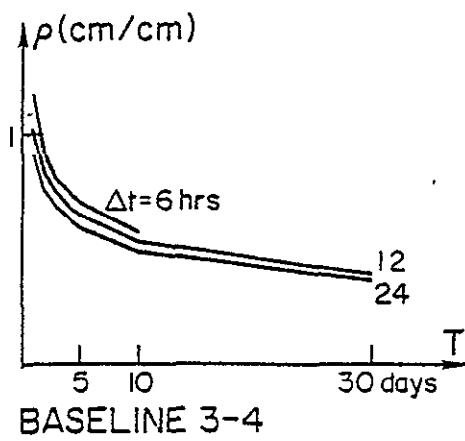


Fig. 9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2).

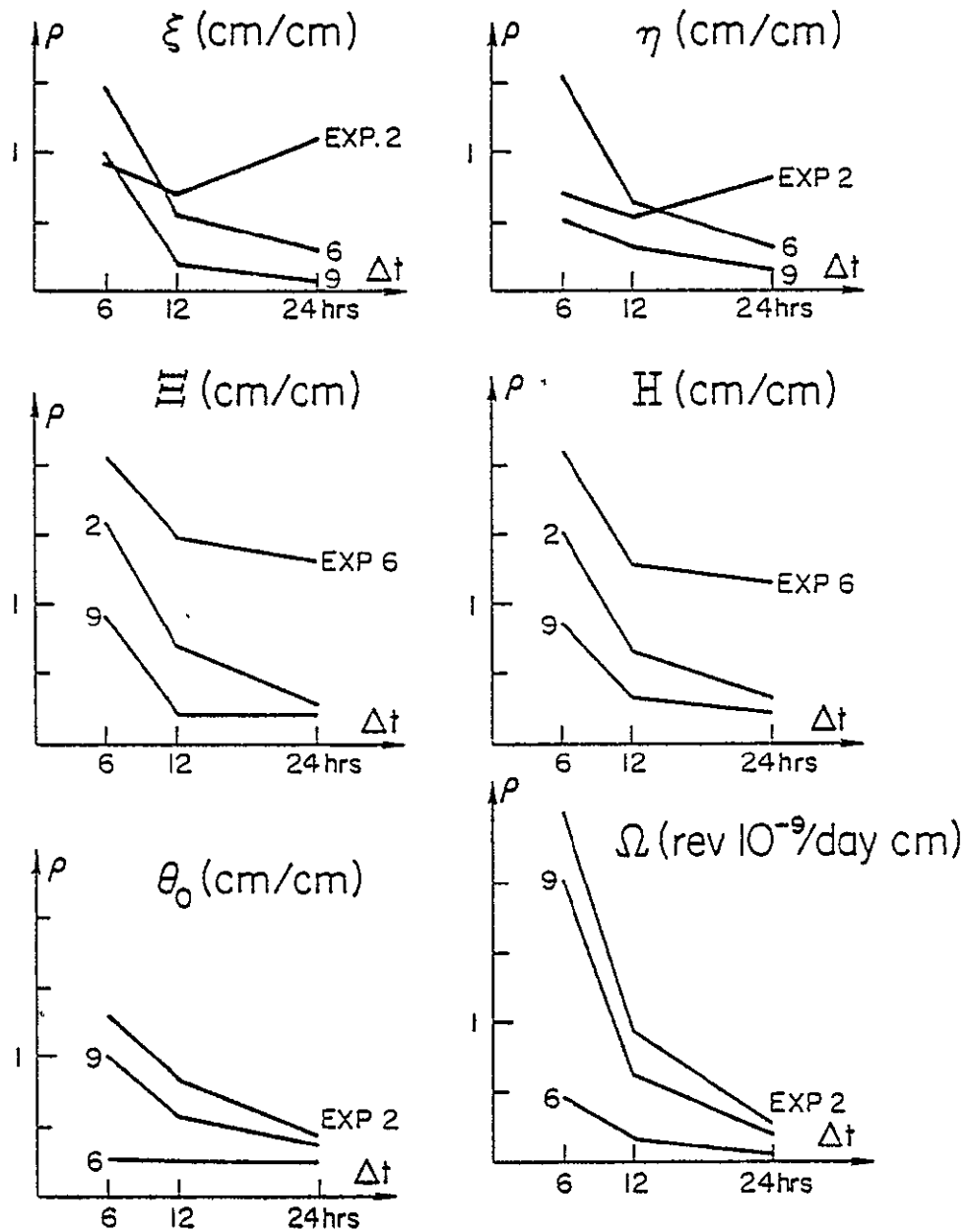


Fig. 9.6 Variation of earth rotation parameter recovery with earth step.

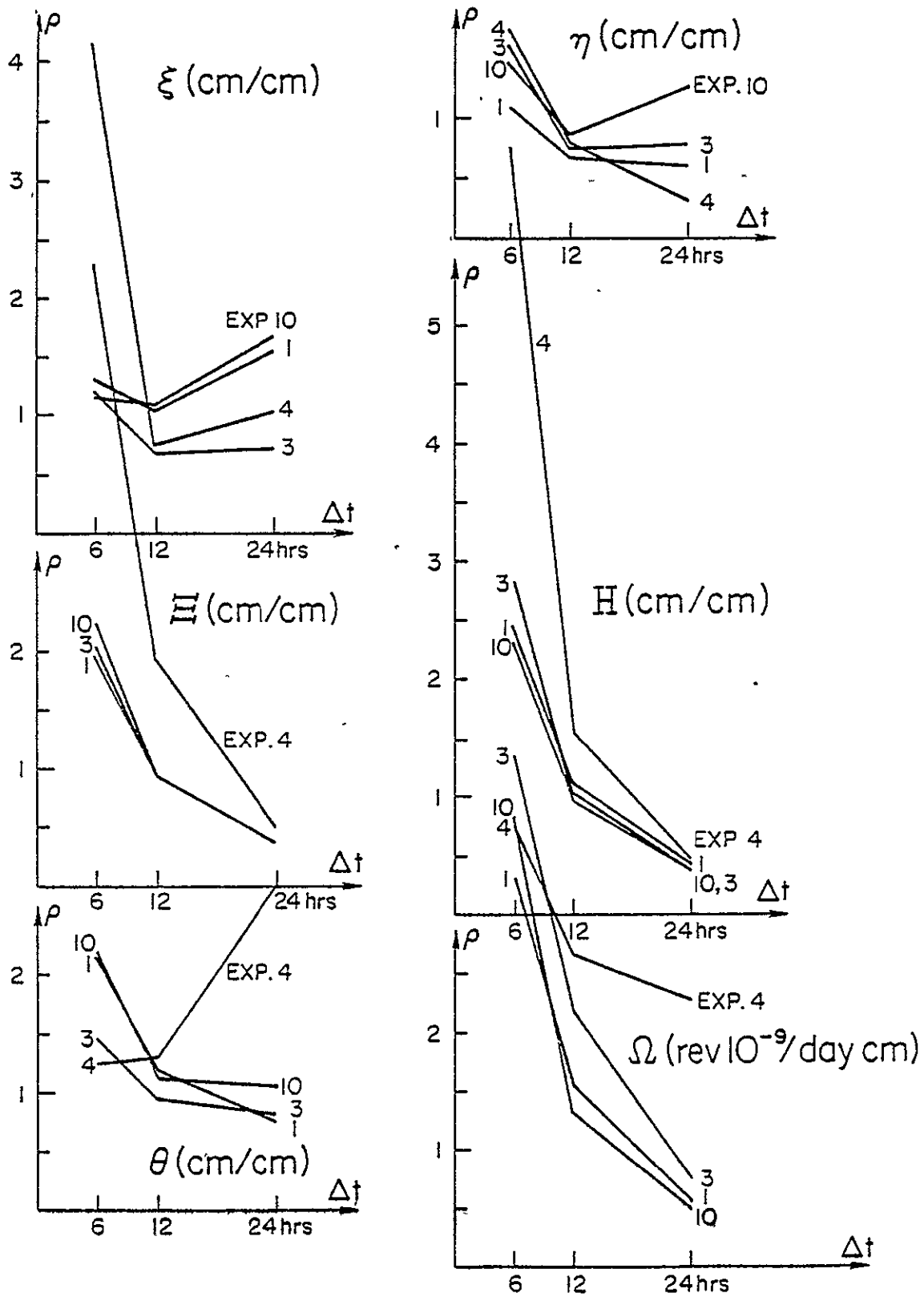


Fig. 9.6 (cont'd)

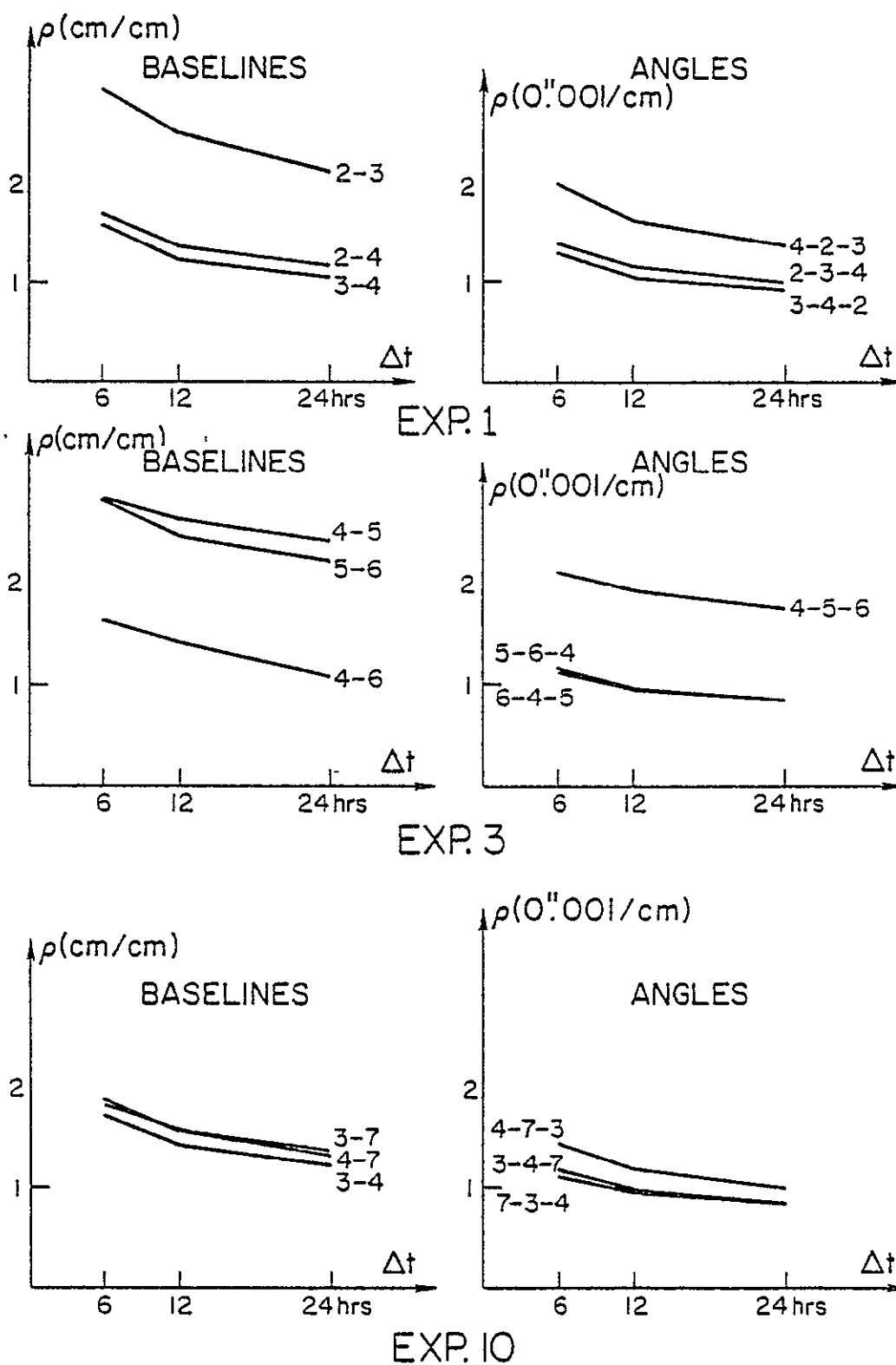


Fig. 9.7 Variation of baseline length and angle recovery with earth step.

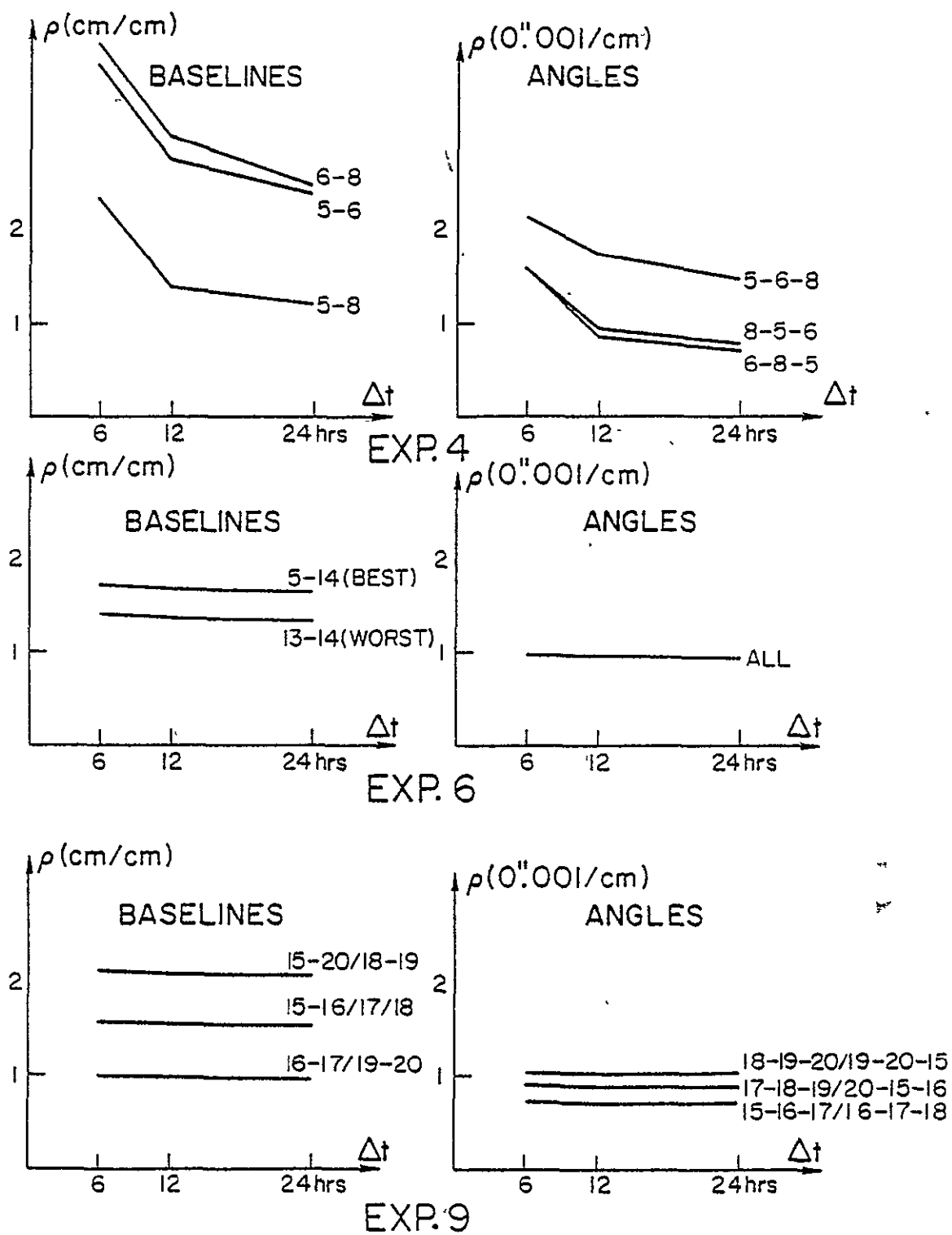


Fig. 9.7 (cont'd)

References

- Arnold, K. (1974). "Geodetic Aspects of Laser Distance Measurements to the Moon and Radio-Interference Measurements to Quasars," Gerlands Beiträge zur Geophysik, 83, 4, 249-269.
- Blaha, G. (1971). "Inner Adjustment Constraints with Emphasis on Range Observations," Reports of the Department of Geodetic Science, No. 148, The Ohio State University, Columbus.
- Grafarend, E. (1974). "Optimization of Geodetic Networks," The Canadian Surveyor, 28, 5, 716-723.
- Koopmans, L.H. (1974). The Spectral Analysis of Time Series. Academic Press, New York.
- Mueller, I.I. (1969). Spherical and Practical Astronomy as Applied to Geodesy. Frederick Ungar Publishing Co., New York.
- Pope, A.J. (1971). "Transformation of Covariance Matrices Due to Changes in Minimal Control." Paper presented at the AGU Annual Fall Meeting, San Francisco.
- Shapiro, I.I. and C.A. Knight. (1970). "Geophysical Applications of Long-Baseline Radio Interferometry." In: L. Mansinha et al. (eds.), Earthquake Displacement Fields and the Rotation of the Earth, pp. 284-301. D. Reidel Publishing Co., Dordrecht, Holland.
- Thomas, J.B. (1972a). "An Analysis of Long Baseline Radio Interferometry," JPL Technical Report 32-1526, VII, pp. 37-50.
- Thomas, J.B. (1972b). "An Analysis of Long Baseline Radio Interferometry, Part II." JPL Technical Report 32-1526, VIII, pp. 29-38.
- Thomas, J.B. (1973). "An Analysis of Long Baseline Radio Interferometry, Part III." JPL Technical Report 32-1526, XVI, p. 47.
- Uotila, U.A. (1967). "Introduction to Adjustment Computations with Matrices." Lecture Notes, Department of Geodetic Science, The Ohio State University, Columbus.

APPENDIX A

Inner Constraints and Their Relation to the "Geometry" of the Pseudoinverse of an Operator

Suppose that a system of linear equations

$$\begin{array}{ccc} N & x & = y \\ u \times u & u \times 1 & u \times 1 \end{array}$$

is given (N, y known; x unknown) which is consistent (i.e., it has at least one solution), and furthermore it has more than one solution. The objective is to show that there exists a set of linear constraints

$$E^T x = 0$$

called inner constraints, such that the solution to the simultaneous system of equations $Nx = y$ and $E^T x = 0$ is unique and identical to the solution $x_0 = N^+ y$, where N^+ is the pseudoinverse of the matrix N .

The matrix N is a representation of an operator $N: X \rightarrow Y$, where X and Y are both u -dimensional inner product spaces with elements $u \times 1$ vectors and the usual Euclidean inner product

$$\langle f, g \rangle = g^T f \quad f, g \in X \quad \text{or} \quad f, g \in Y$$

Two subspaces can be introduced with respect to the operator N , the range $R(N) = R$ of N defined as

$$R = \{y; y \in Y \text{ and } y = Nx \text{ for some } x \in X\}, \quad R \subset Y$$

and the kernel or null space $K(N) = K \subset X$ of N ,

$$K = \{x; x \in X \text{ and } Nx = 0\}$$

The consistency of the linear equations $Nx = y$ refers to the fact that $y \in R$, while the non-uniqueness of solution to the fact that $K \cap \{0\}^c$ is non-empty, i.e., K has other elements in addition to 0. If x' is a solution to $Nx = y$, then $x = x' + \bar{x}$ is also a solution where \bar{x} is any element of K . We can therefore identify the solution

space $S_y \subset X$, defined as

$$S_y = \{x; x \in X \text{ and } Nx = y, y \in R \text{ fixed}\}$$

with a linear variety

$$S_y = \{x; x = x' + \bar{x}, \bar{x} \in K, x' \text{ fixed with } Nx' = y\}$$

If K^\perp is the orthogonal complement of K with respect to X , the projection $x_0 = \mathcal{P}_{K^\perp}(x)$ of any element $x \in S_y$ on K^\perp is unique, and furthermore [Dermanis, 1977, §3.5]

$$\|x_0\| = \min_{x \in S_y} \|x\|$$

We can now define the pseudoinverse N^+ of the operator N as an operator $N^+: R \rightarrow X$ where

$$N^+ y = x_0 = \mathcal{P}_{K^\perp}(x) \quad \text{for any } y \in R \text{ and } x \in S_y$$

N^+ is therefore the ordinary inverse of the restriction of the operator N to K^\perp (see [Desoer and Whalen, 1963, p. 444] for a more rigorous definition). The domain of definition of N^+ can be extended to the whole space Y ($N^+: Y \rightarrow X$) by setting

$$N^+(y) = N^+(y')$$

where $y' = \mathcal{P}_R(y)$ is the projection of $y \in Y$ on R .

The range of N^+ is K^\perp , while its kernel (null) space is R^\perp . For $y \in R$ we have $x_0 = N^+ y$ and x_0 can be alternatively defined as the unique element in the intersection $S_y \cap K^\perp$. This means that x_0 is uniquely defined by its two properties:

$$x_0 \in S_y \quad \text{and} \quad x_0 \in K^\perp$$

The first property is expressed by the original equations $Nx = y$, while a similar expression must be established for the second property. Let $r = \text{rank}(N) < u$ and $s = u - r$ be the rank deficiency of N . The dimension of the linear subspace $K \subset X$ is then s , and suppose that $\{e_i\}$, $i = 1, 2, \dots, s$ is a basis in K . Since $x_0 \in K^\perp$, we have $x_0 \perp e_i$ for $i = 1, 2, \dots, s$, i.e.,

$$\langle x_0, e_i \rangle = e_i^T x_0 = 0 \quad \text{for } i = 1, 2, \dots, s$$

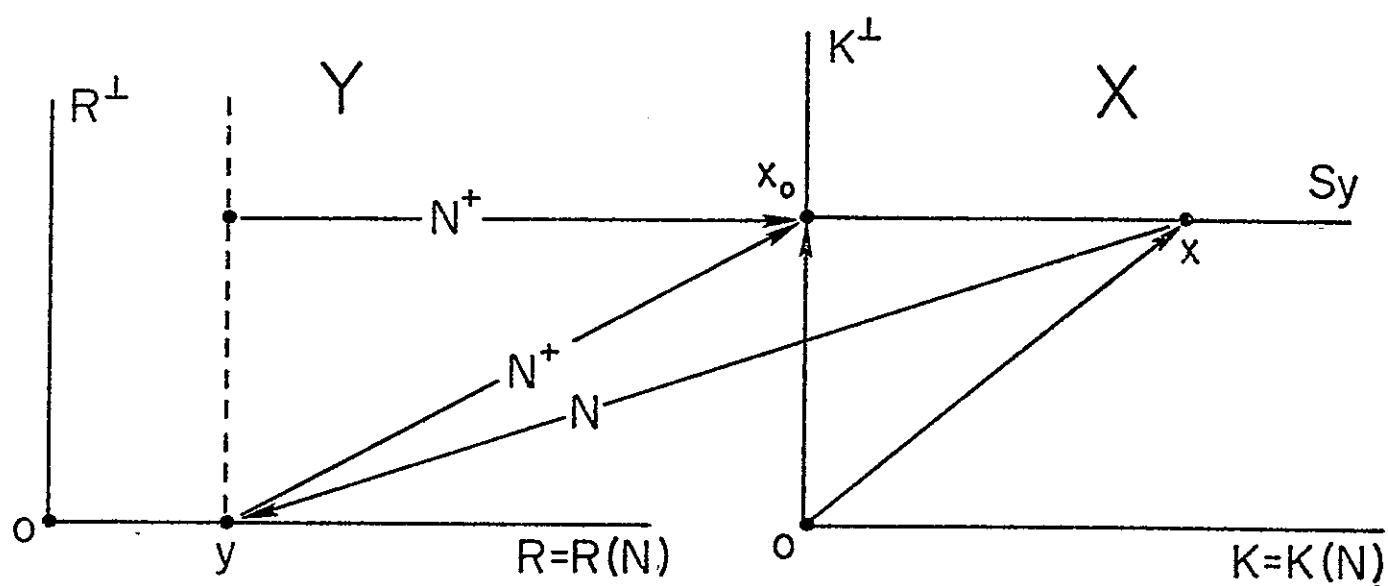


Fig. A.1 The geometry of the pseudoinverse operator.

Setting

$$\begin{matrix} E & = & [e_1 & e_2 & \dots & e_s] \\ u \times s \end{matrix}$$

the above equations can be written in the compact matrix form

$$E^T x_0 = 0$$

Therefore, x_0 is uniquely determined as the vector satisfying both equations

$$Nx = y \quad \text{and} \quad E^T x = 0$$

where the columns of E constitute a basis in K , the kernel (null space) of N . The constraints $E^T x = 0$ are referred to as the "inner" constraints.

The problem has now been reduced to that of finding a basis in K , i.e., of finding a set of s linearly independent $u \times 1$ vectors $\{e_i\}$ satisfying

$$N e_i = 0 \quad \text{for } i = 1, 2, \dots, s$$

The equations above can be written again in matrix form

$$N E = 0 \quad \text{where} \quad E = [e_1 \ e_2 \ \dots \ e_s]$$

We therefore only need to find a $u \times s$ matrix E with $\text{rank}(E) = s = u - \text{rank}(N)$ (to secure linear independence of its columns e_i) satisfying $NE = 0$.—Summarizing, we have the following theorem

Given the system of linear equations

$$\begin{matrix} Nx & = & y & & (N, y \text{ known; } x \text{ unknown}) \\ u \times u \times 1 & & u \times 1 \end{matrix}$$

where $\text{rank}(N) = r < u$, $s = u - r$, and a $u \times s$ matrix E with $\text{rank}(E) = s$ satisfying $NE = 0$, the vector x satisfying simultaneously $Nx = y$ and $E^T x = 0$ is unique and identical to $x = N^+ y$, where N^+ is the pseudoinverse of N .

References

- Dermanis, A. 1977. "Geodetic Linear Estimation Techniques and the Norm Choice Problem," Manuscripta Geodactica, 2, 1.
- Desoer, C.A and B.H. Whalen. 1963. "A Note on Pseudoinverses," Journal of the Society for Industrial and Applied Mathematics, 11, 2, 442-447.

APPENDIX B

Algorithm for First-Order Partitioned Linear Regression Including Inner Constraints

The problem in question is the construction of an algorithm for the least squares adjustment of a group of linearized observation equations of the form

$$V_i = \ddot{A}_i \ddot{X}_i + \dot{A}_i \dot{X} + L_i \quad i = 1, 2, \dots, m \quad (A \ 1)$$

with $E\{V_i\} = 0$ and $E\{V_i V_i^T\} = P_i$. \dot{X} refers to parameters common to all sets of observations, while \ddot{X}_i refers to those appearing only in the i^{th} set of observations. Each set of observation equations gives rise to a contribution to the normal equations of the form

$$\begin{bmatrix} \ddot{N}_i & \ddot{N}_i^T \\ \ddot{N}_i^T & \dot{N}_i \end{bmatrix} \begin{bmatrix} \ddot{X}_i \\ \dot{X} \end{bmatrix} + \begin{bmatrix} \ddot{U}_i \\ \dot{U}_i \end{bmatrix} = 0 \quad (A \ 2)$$

where

$$\begin{aligned} \ddot{N}_i &= \ddot{A}_i^T P_i \ddot{A}_i & \ddot{U}_i &= \ddot{A}_i^T P_i L_i \\ \dot{N}_i &= \dot{A}_i^T P_i \dot{A}_i & \dot{U}_i &= \dot{A}_i^T P_i L_i \end{aligned}$$

and

$$\ddot{N}_i = \ddot{A}_i^T P_i A_i$$

The combination of such individual sets of normal equations leads to the final set of normal equations incorporating all observations

$$\begin{bmatrix} \ddot{N} & \ddot{N}^T \\ \ddot{N}^T & \dot{N} \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} \ddot{U} \\ \dot{U} \end{bmatrix} = 0 \quad (A \ 3)$$

where

$$\ddot{N} = \begin{bmatrix} \ddot{N}_1 & 0 & & 0 \\ 0 & \ddot{N}_2 & & \\ & & \ddots & \\ 0 & & & \ddot{N}_m \end{bmatrix}, \quad \ddot{N} = \begin{bmatrix} \ddot{N}_1 \\ \ddot{N}_2 \\ \vdots \\ \ddot{N}_m \end{bmatrix}, \quad \ddot{U} = \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \\ \vdots \\ \ddot{U}_m \end{bmatrix}, \quad \ddot{X} = \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \vdots \\ \ddot{X}_m \end{bmatrix}$$

$$\dot{\mathbf{N}} = \sum_i \dot{\mathbf{N}}_i, \quad \dot{\mathbf{U}} = \sum_i \dot{\mathbf{U}}_i$$

In case $\mathbf{N} = \begin{bmatrix} \ddot{\mathbf{N}} & \overline{\mathbf{N}} \\ \overline{\mathbf{N}}^T & \dot{\mathbf{N}} \end{bmatrix}$ is nonsingular ($\det \mathbf{N} \neq 0$), the normal equations above can

be solved through a First-Order Partitioned Linear Regression scheme as explained in [Brown and Trotter, 1969; and Uotila, 1973]. We give here an extension of this scheme for the case when \mathbf{N} is singular and a unique solution is ascertained only after the introduction of a set of inner constraints

$$\ddot{\mathbf{E}}^T \ddot{\mathbf{X}} + \dot{\mathbf{E}}^T \dot{\mathbf{X}} = 0 \quad (\text{A } 4)$$

with

$$\ddot{\mathbf{E}}^T = [\ddot{\mathbf{E}}_1^T \ \ddot{\mathbf{E}}_2^T \ \dots \ \ddot{\mathbf{E}}_m^T] = [\ddot{\mathbf{E}}_0^T \ \ddot{\mathbf{E}}_0^T \ \dots \ \ddot{\mathbf{E}}_0^T]$$

The normal equations can now be augmented as follows [Uotila, 1967]:

$$\begin{bmatrix} \ddot{\mathbf{N}} & \overline{\mathbf{N}} & \ddot{\mathbf{E}} \\ \overline{\mathbf{N}}^T & \dot{\mathbf{N}} & \dot{\mathbf{E}} \\ \ddot{\mathbf{E}}^T & \dot{\mathbf{E}}^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{X}} \\ \dot{\mathbf{X}} \\ \mathbf{K}_L \end{bmatrix} + \begin{bmatrix} \ddot{\mathbf{U}} \\ \dot{\mathbf{U}} \\ 0 \end{bmatrix} = 0 \quad (\text{A } 5)$$

The new augmented coefficient matrix is nonsingular and inversion with the introduction of some new notation leads to

$$\begin{bmatrix} \ddot{\mathbf{N}} & \overline{\mathbf{N}} & \ddot{\mathbf{E}} \\ \overline{\mathbf{N}}^T & \dot{\mathbf{N}} & \dot{\mathbf{E}} \\ \ddot{\mathbf{E}}^T & \dot{\mathbf{E}}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \ddot{\mathbf{M}} & \overline{\mathbf{M}} \\ \overline{\mathbf{M}}^T & \dot{\mathbf{M}} \end{bmatrix}^{-1} = \begin{bmatrix} \ddot{\mathbf{Q}} & \overline{\mathbf{Q}} \\ \overline{\mathbf{Q}}^T & \dot{\mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \ddot{\mathbf{G}} & \overline{\mathbf{G}} & \ddot{\mathbf{F}} \\ \overline{\mathbf{G}}^T & \dot{\mathbf{G}} & \dot{\mathbf{F}} \\ \ddot{\mathbf{F}}^T & \dot{\mathbf{F}}^T & \mathbf{F} \end{bmatrix} \quad (\text{A } 6)$$

With the help of this inverse, the solution of the normal equations becomes

$$\begin{aligned} -\dot{\mathbf{X}} &= \ddot{\mathbf{G}} \ddot{\mathbf{U}} + \overline{\mathbf{G}} \dot{\mathbf{U}} \\ -\ddot{\mathbf{X}} &= \overline{\mathbf{G}}^T \ddot{\mathbf{U}} + \dot{\mathbf{G}} \dot{\mathbf{U}} \end{aligned} \quad (\text{A } 7)$$

If $\overline{\mathbf{G}}_1$, $\ddot{\mathbf{G}}_{1j}$ denote submatrices of $\overline{\mathbf{G}}$ and $\ddot{\mathbf{G}}$, respectively, with dimensions equal to those of $\overline{\mathbf{N}}_1$ and $\ddot{\mathbf{N}}_{1j}$, respectively, we have

$$\begin{aligned} -\dot{\mathbf{X}} &= \sum_j \overline{\mathbf{G}}_j^T \ddot{\mathbf{U}}_j + \dot{\mathbf{G}} \mathbf{U} \\ -\ddot{\mathbf{X}}_1 &= \sum_j \ddot{\mathbf{G}}_{1j} \ddot{\mathbf{U}}_j + \overline{\mathbf{G}}_1 \dot{\mathbf{U}} \end{aligned} \quad (\text{A } 8)$$

The inversion of Eq. (A 6) leads to

$$\begin{aligned}
\dot{\bar{Q}} &= [\ddot{\bar{M}} - \bar{M}^T \ddot{\bar{M}}^{-1} \bar{M}]^{-1} \\
\bar{Q} &= -\ddot{\bar{M}}^{-1} \bar{M} \dot{\bar{Q}} \\
\ddot{\bar{Q}} &= \ddot{\bar{M}}^{-1} + \ddot{\bar{M}}^{-1} \bar{M} \dot{\bar{Q}} \bar{M}^T \ddot{\bar{M}}^{-1}
\end{aligned} \tag{A 9}$$

Replacing $\ddot{\bar{M}}$, $\dot{\bar{M}}$, \bar{M} from (A 6) in terms of submatrices of $\ddot{\bar{N}}$, $\dot{\bar{N}}$, \bar{N} , and introducing the following notation

$$\begin{aligned}
H_1 &= \ddot{N}_1^{-1} \bar{N}_1 & R_1 &= N_1^T H_1 \\
S_1 &= N_1 - R_1 & S &= \sum_1 S_1 \\
H &= \sum_1 H_1 & K &= \sum_1 \ddot{N}_1^{-1} \\
\bar{L}_1 &= H_1^T \ddot{U}_1 & L &= \sum_1 \bar{L}_1 \\
T_1 &= \ddot{N}_1^{-1} \ddot{U}_1 & T &= \sum_1 T_1
\end{aligned}$$

we finally obtain after some algebraic manipulations

$$\dot{\bar{Q}}^{-1} = \begin{bmatrix} S & (\dot{\bar{E}} - H^T \ddot{\bar{E}}_0) \\ (\dot{\bar{E}}^T - \ddot{\bar{E}}_0^T H) & (-\ddot{\bar{E}}_0^T K \ddot{\bar{E}}_0) \end{bmatrix} \tag{A 10}$$

$$\dot{\bar{X}} = \dot{\bar{G}} L + \dot{\bar{F}} \ddot{\bar{E}}_0^T T - \dot{\bar{G}} \ddot{U} \tag{A 11}$$

$$\ddot{\bar{X}}_1 = -T_1 - \ddot{N}_1^{-1} [\ddot{\bar{E}}_0 \dot{\bar{F}}^T (-\ddot{U} + L) + (\ddot{\bar{E}}_0 F \ddot{\bar{E}}_0^T) T] - H_1 \dot{\bar{X}} \tag{A 12}$$

$$\begin{aligned}
\ddot{G}_{1j} &= \delta_{1j} \ddot{N}_1^{-1} + H_1 \dot{\bar{G}} H_j^T + H_1 (F \ddot{\bar{E}}_0^T) \ddot{N}_j^{-1} + \\
&+ \ddot{N}_1^{-1} (\ddot{\bar{E}}_0 \dot{\bar{F}}^T) H_j^T + \ddot{N}_1^{-1} (\ddot{\bar{E}}_0 F \ddot{\bar{E}}_0^T) \ddot{N}_j^{-1}
\end{aligned} \tag{A 13}$$

The algorithm can now be summarized in the following steps:

1. Compute $\ddot{A}_i, \dot{A}_i, L_i; \ddot{E}, \ddot{E}_0$
2. Compute $\ddot{N}_i, \dot{N}_i, \ddot{U}_i, \dot{U}_i$
3. Compute $H_i, R_i, S_i, T_i, \bar{L}_i$
4. Compute $H, S, \dot{N}, \dot{U}, K, L, T$
5. Obtain \dot{G}, \dot{F}, F by inversion

$$\begin{bmatrix} S & E - H^T \ddot{E}_0 \\ (\dot{E} - H^T \ddot{E}_0)^T & -\ddot{E}_0^T K \ddot{E}_0 \end{bmatrix}^{-1} = \begin{bmatrix} \dot{G} & \dot{F} \\ \dot{F}^T & F \end{bmatrix}$$

6. Compute \dot{X} from (A 11), \ddot{X}_i from (A 12), and \ddot{G}_{ij} from (A 13)

References

Brown, D.C. and J.E. Trotter. 1969. "SAGA, A Computer Program for Short Arc Geodetic Adjustment of Satellite Observations." DBA Systems Inc., Melbourne, Florida.

Uotila, U.A. 1973. "First-Order Partitioned Regression." Unpublished Manuscript.

ORIGINAL PAGE IS
OF POOR QUALITY

APPENDIX C

Computer Programs

The simulation of observations and their adjustment are carried out with the help of three programs, VLBI SIMULATOR, PROGØ1, and PROGØ2 as explained in Chapter 7. These programs are presented here in detail together with their supporting subroutines.

1. Name of Program: VLBI SIMULATOR

Function: Simulates distance and distance rate VLBI observations corresponding to time delays and their derivatives, according to a previously decided observational pattern.

Input: Number of participating stations and radio sources, radio source and station coordinates, initial and final epochs of observations, time interval between successive observations, identifying numbers of stations and radio source participating in each observation.

Output: Identification numbers for participating stations and radio source, observed (perfect) distance and distance rate, epoch of observation.

Subroutines required:

GRESID: Calculates Greenwich sidereal time for a given epoch

JULIA: Converts Universal Time to Julian date

EARTH: Provides simulated earth rotation parameters for a given epoch

REDPI: Converts an angle in radians to the $(-\pi, \pi)$ interval.

The following is a listing of VLBI SIMULATOR program and supporting subroutines.

```

C***** CONVERT RA AND DEC TO RADIANs
0011      RAR=RA(I)*PI/180.DO
0012      DECR=DEC(I)*PI/180.DO
0013      CD=DCOS(DECR)
0014      SD=DSIN(DECR)
0015      CA=DCOS(RAR)
0016      SA=DSIN(RAR)
0017      F1(I)=CD*CA
0018      E2(I)=CD*SA
0019      F3(I)=SD
0020      5      CONTINUE
C
C
C***** READ STATION COORDINATES
C***** PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
0021      DO 6 I=1,IN
0022      READ(5,701) IDUM,IL1,IL2, LONG(I), PHI(I), R(I)
0023      701  FORMAT(1X,I2,1X,A4,A2,3F15.1)
0024      WRITE(6,701) I,IL1,IL2, LONG(I), PHI(I), R(I)
0025      101  FORMAT(3F15.5)
C***** CONVERT PHI AND LONG TO RADIANs
0026      PHIR=PHI(I)*PI/180.DO
0027      LONGR=LONG(I)*PI/180.DO
0028      CF=DCOS(PHIR)
0029      SF=DSIN(PHIR)
0030      CL=DCOS(LONGR)
0031      SL=DSIN(LONGR)
0032      X(I)=CF*CL
0033      Y(I)=CF*SL
0034      Z(I)=SF
0035      6      CONTINUE
C
C
0036      DTG=PI/12.DO
C
C***** READ INITIAL EPOCH IN UT
0037      READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
0038      WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
0039      201  FORMAT(1X,I4,4I5,F15.1)
C
C
C***** FIND GREENWICH SIDERIAL TIME AT EPOCH TO
0040      CALL GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
C
C
C***** READ FINAL EPOCH TF AND TIME INTERVAL INHOURS
0041      READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0042      WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0043      READ(5,101) DT
0044      WRITE(6,101) DT
C
C
0045      IF(JYEAR.NE.IYEAR) GO TO 88
0046      IF(JMO.NE.IMO) GO TO 88
C***** NOTICE THAT IF TO, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
C***** DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
0047      79  TF=DFLOAT(JDAY-IDAY)*24.DO+DFLOAT(JHOUR-IHOUR)+DFLOAT(JMIN-IMIN)

```

ORIGINAL PAGE IS
OF POOR QUALITY

7/60.D0+(SECJ-SEC)/3600.D0

C
C

0048 T0=0.D0
0049 T=0.D0
0050 33 IF(T.GT.TF) GO TO 40

C

0051 DO 8 I=1,IM
C***** TRANSFORM QUASAR UNIT VECTOR TO EARTH FIXED SYSTEM AT EPOCH T
0052 ANGLE=OMG*(T-T0)+TH0
0053 CA=DCOS(ANGLE)
0054 SA=DSIN(ANGLE)
0055 Q1=CA*E1(I)+SA*E2(I)
0056 Q2=-SA*E1(I)+CA*E2(I)
0057 Q3=E3(I)

C

0058 DO 8 J=1,IN
0059 ZNT=PARCOS(X(J)*Q1+Y(J)*Q2+Z(J)*Q3)
0060 ARGU=X(J)*Q1+Y(J)*Q2+Z(J)*Q3
0061 IF(ARGU.LT.-1.D0) ARGU=-1.D0
0062 IF(ARGU.GT.1.D0) ARGU=1.D0
0063 ZNT=PARCOS(ARGU)
0064 ZNT=DARS(ZNT)*180.D0/PI
0065 ZNTMAX=60.D0
0066 IF(ZNT.GT.ZNTMAX) ZNT=10.D0**10
0067 INDEX(I,J)=ZNT
0068 8 CONTINUE

C

0069 WRITE(6,207) T
0070

0071 207 FORMAT('1'//10X,'EPOCH = ',F15.5//20X,'1',2X,'2',2X,'3',2X,

0072 '4',2X,'5',2X,'6',2X,'7',2X,'8',2X,'9',1X,'10',1X,'11',1X,'12',

0073 '13',1X,'14',1X,'15',1X,'16',1X,'17',1X,'18',1X,'19',1X,'20',
'21',1X,'22',1X,'23',1X,'24',1X,'25',1X,'26',1X,'27',1X,'28',
'29',1X,'30'//)

C

0074 DO 9 I=1,IM
0075 WRITE(6,105) I,RA(I),DEC(I),(INDEX(I,J),J=1,IN)
0076 105 FORMAT(1X,I2,2F7.0,1X,30(1X,I2))
0077 CONTINUE

C

C

0078 INM=IN-1
0079 DO 555 I=1,IM
0080 DO 555 J=1,INM
0081 IF(INDEX(I,J).GT.1000) GO TO 555
0082 JP=J+1
0083 DO 555 K=JP,IN

```

0084      IF(INDEX(I,K).GT.1000) GO TO 555
0085      WRITE(7,705) J,K,I,T
0086      705  FORMAT(3I5,F15.2)
0087      555  CONTINUE
          C
          C
0088      99   T=T+DT
0089      GO TO 33
0090      88   WRITE(6,500)
0091      500   FORMAT('1'////////10X,'WRONG INITIAL AND FINAL DATE'/
0092           710X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
0093      40   STOP
          END

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
C***** RA AND DEC ARE IN DEGREES
0008      DO 5 I=1,IM
0009          READ(5,321) IDM,IL1,IL2,IL3,RA(I),DEC(I)
0010          WRITE(6,321) I,IL1,IL2,IL3,RA(I),DEC(I)
0011      321  FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
C***** CONVFT RA AND DEC TO RADIANS
0012      RAR=RA(I)*PI/180.DO
0013      DECR=DEC(I)*PI/180.DO
0014      CD=DCOS(DECR)
0015      SD=DSIN(DECR)
0016      CA=DCOS(RAR)
0017      SA=DSIN(RAR)
0018      E1(I)=CD*CA
0019      E2(I)=CD*SA
0020      E3(I)=SD
0021      5  CONTINUE
C
C
C***** READ STATION COORDINATES
C***** PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
0022      DO 6 I=1,IN
0023          READ(5,701) IDM,IL1,IL2,LONG(I),PHI(I),R(I)
0024          WRITE(6,701) I,IL1,IL2,LONG(I),PHI(I),R(I)
0025      701  FORMAT(1X,I2,1X,A4,A2,3F15.1)
C***** CONVERT PHI AND LONG TO RADIANS
0026      PHIR=PHI(I)*PI/180.DO
0027      LONGR=LONG(I)*PI/180.DO
0028      CF=DCOS(PHIR)
0029      SF=DSIN(PHIR)
0030      CL=DCOS(LONGR)
0031      SL=DSIN(LONGR)
0032      X(I)=CF*CL
0033      Y(I)=CF*SL
0034      Z(I)=SF
0035      6  CONTINUE
C
C
C***** READ CORRECTION TO GREENWITCH SIDERIAL TIME DDTH IN ARC SEC
0036      READ(5,101) DDTH
0037      WRITE(6,101) DDTH
0038      101  FORMAT(3F15.5)
C***** CONVERT TO RADIANS
0039      DDTH=DDTH*PI/(180.DO*3600.DO)
C
C
C***** READ INITIAL EPOCH IN UT
0040      READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
0041      WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
0042      201  FORMAT(5I5,F15.5)
C
C
C***** FIND GREENWITCH SIDERIAL TIME AT EPOCH TO
0043      CALL GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
0044      COPY(1)=THO
0045      THO=THO*(180.DO/PI)
0046      WRITE(6,136) THO
0047      136  FORMAT(///10X7'APPR. THO= ',F25.9//)
0048      WRITE(7,101) THO
```



```

0049      TH0=COPY(1)+DDTH
          C
          C
          C***** READ FINAL EPOCH TF AND TIME INTERVAL INHOURS
0050      READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0051      WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0052      READ(5,101) DT
0053      WRITE(6,101) DT
          C
          C
0054      IF(JYEAR.NE.IYEAR) GO TO 88
0055      IF(JMO.NE.IMO) GO TO 88
          C***** NOTICE THAT IF TO, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
          C***** DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
0056      79 TF=DFLOAT(JDAY-IDAY)*24.00+DFLOAT(JHOUR-1HOUR)+DFLOAT(JMIN-1MIN)
          77/60.00+(SECJ-SEC)/3600.00
          C
          C
0057      TO=0.00
0058      T=0.00
0059      33 IF(T.GT.TF) GO TO 40
          C
0060      555 READ(5,107) I,J,K
0061      IF(I.EQ.0) GO TO 99
          C***** I=0 IS A CODE TO INDICATE END OF OBSERVATIONS FOR THIS EPOCH
          C
0062      CALL EARTH(T,XP,HP,OMG,THETA,XX,HH)
          C
          C
          C
0063      XPD=XP*REART
0064      HPD=HP*RFART
0065      XXD=XX*REART
0066      HHD=HH*RFART
0067      OMGD=OMG*(12.00/PI)
0068      THET=THETA
0069      THET=THET*(180.00/PI)
0070      ITH1=THET
0071      TH2=(THET-DFLOAT(ITH1))*60.00
0072      ITH2=TH2
0073      TH3=(TH2-DFLOAT(ITH2))*60.00
0074      WRITE(6,801) T,XPD,HPD,XXD,HHD,ITH1,ITH2,TH3,OMGD
0075      801 FORMAT(3X,F7.2,4F10.4,1X,2I4,1X,F8.4,F12.9)
          C
0076      COPY(1)= E1(K) -XX*E3(K)
0077      COPY(2)= + E2(K)+HH*E3(K)
0078      COPY(3)=XX*E1(K)-HH*E2(K)+ E3(K)
          C
0079      ANGLE=THETA+TH0
0080      CA=DCOS(ANGLE)
0081      SA=DSIN(ANGLE)
          C
0082      Q1= CA*COPY(1)+SA*COPY(2)
0083      Q2=-SA*COPY(1)+CA*COPY(2)
0084      Q3=COPY(3)
          C
0085      QF1=-SA*COPY(1)+CA*COPY(2)
0086      QF2=-CA*COPY(1)-SA*COPY(2)
0087      QF3=-XP*QF1+HP*QF2

```

```
C
0088      COPY(1)=  Q1      +XP*Q3
0089      COPY(2)=      Q2-HP*Q3
0090      COPY(3)=-XP*Q1+HP*Q2+  Q3
C
0091      DS=(X(J)*R(J)-X(I)*R(I))*COPY(1)+(Y(J)*R(J)-Y(I)*R(I))*COPY(2)
        ? +(Z(J)*R(J)-Z(I)*R(I))*COPY(3)
C
0092      FRNG=(X(J)*R(J)-X(I)*R(I))*QF1+(Y(J)*R(J)-Y(I)*R(I))*QF2
        2+(Z(J)*R(J)-Z(I)*R(I))*QF3
C
0093      FRNG= DERIVATIVE OF DELAY DISTANCE DS (METERS/POUR)
        FRNG=FRNG*Q1G
C
0094      WRITE(6,107) I,J,K,DS,FRNG,T
0095      WRITE(7,107) I,J,K,DS,FRNG,T
0096      107  FORMAT(3I5,3F15.5)
0097      GO TO 555
C
0098      99  T=T+DT
0099      GO TO 33
0100      88  WRITE(6,500)
0101      500  FORMAT('1'////////10X,'WRONG INITIAL AND FINAL DATE'/
        ?10X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
0102      40  STOP
0103      END
```

ORIGINAL PAGE IS
OF POOR QUALITY


```

0001      SUBROUTINE GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
C*****
C*****
C*****
C*****      CALCULATES THE GREENWICH SIDERIAL TIME
C*****      THO, FOR EPOCH WITH DATE IYEAR IMO IDAY
C*****      IHOUR IMIN SEC UT.
C*****
C*****
C*****
0002      IMPLICIT REAL*8(A-H,L-Z)
0003      PI=4.00*DATAN(1.00)
0004      I1=0
0005      I2=0
0006      A=0.00
0007      CALL JULIA(IYEAR,IMO,IDAY,I1,I2,A,MJD)
0008      T=(MJD-2415020.00)/36525.00
0009      T2=T*T
0010      THO=99.69098320+3.6000.768900*T+0.0003870800*T2
0011      THO=THO*PI/180.00
0012      CALL REDPI(THO)
0013      DT=DFLOAT(IHOUR)*60.00+DFLOAT(IMIN)+SEC/60.00
0014      DTHDT=4.375269500/(10.00**3)
0015      THO=THO+DT*DTHDT
0016      CALL REDPI(THO)
0017      RETURN
0018      END

```

0001

SUBROUTINE JULIA(IYFAR,IM,IDAY,IHHH,IMMM,S,MJD)

```

C*****
C*****
C*****
C***** SUBROUTINE TO CONVERT UNIVERSAL TIME TO
C***** JULIAN DATE
C*****
C*****
C*****

```

0002

IMPLICIT REAL*8 (A-H,L-Z)

0003

DIMENSION IMONTH(12)

0004

DATA IMONTH /0,31,59,90,120,151,181,212,243,273,304,334/

0005

H=DFLOAT(IHHH)

0006

M=DFLOAT(IMMM)

0007

ICOD=0

0008

IDIS=(IYEAR-1897)/4

0009

IF(IYEAR.GT.1900) IDIS=IDIS-1

0010

ICH=4*(IYEAR/4)

0011

IF(IYEAR.EQ.ICH.AND.IM.GT.2) ICOD=1

0012

IF(IYEAR.EQ.1900) ICOD=0

0013

MJD=2415020+(IYEAR-1900)*365.DO+IDIS+IMONTH(IM)+ICOD-0.5DO+IDAY

0014

X=H/24.DO+M/1440.DO+S/86400.DO

0015

RETURN

END

ORIGINAL PAGE IS
OF POOR QUALITY

```

0001      SUBROUTINE EARTH(T,XP,HP,OMG,THETA,XX,HH)
      C
      C      T : TIME INTERVAL AFTER INITIAL EPOCH (HOURS)
      C      XP,HP: POLAR MOTION COORDINATES (RADIAN)
      C      THETA: UT1 ANGLE IN RADIAN
      C      OMG : ROTATIONAL VELOCITY OF EARTH (RADIAN PER HOUR)
      C      XX,HH: PRECESSION-NUTATION SMALL ANGLES (RADIAN)
0002      IMPLICIT REAL*8(A-H,L-Z)
0003      DIMENSION P(10),A(10),B(10),PS(10),F(10)
      C
0004      DATA P/8760.00,4380.00,2190.00,720.00,360.00,
2160.00,24.00,12.00,1.00,0.500/,
3A/1.1600,0.6300,0.2800,0.1200,0.0600,
40.0300,0.00200,0.00100,0.00000,0.00000/,
5B/0.9500,0.7000,0.2700,0.1300,0.0500,
60.0100,0.00400,0.00100,0.00000,0.00000/,
7PS/3.1500,5.2800,1.3200,0.1500,0.0000,0.9900,2.000,
86.1400,7.7500,4.1300/,
9F/1.1300,2.8500,6.1700,4.3500,0.0500,5.1800,3.7500,1.2400,
70.3500,1.6700/
      C
0005      PI=4.00*(ATAN(1.00))
0006      PI2=2.00*PI
0007      REART=6371000.00
0008      M10=6.00
0009      M20=0.00
0010      PC=429.00*24.00
0011      C=DCOS(PI2*T/PC)
0012      S=DSIN(PI2*T/PC)
0013      M1=C*M10-S*M20
0014      M2=S*M10+C*M20
      C
0015      DO 5 I=1,10
0016      ARG=(PI2*T)/P(I)
0017      M1=M1+A(I)*DCOS(ARG+PS(I))
0018      M2=M2+B(I)*DCOS(ARG+F(I))
0019      CONTINUE
      C
0020      XP=-M2/REART
0021      HP=-M1/REART
0022      XX=0.0600+0.0100*DCOS(2.3400+T/24.00)-0.003*DCOS(0.100+T/12.00)
0023      HH=0.0400+0.0200*DCOS(3.1100+T/24.00)-0.00500*DCOS(0.0700+T/12.00)
0024      XX=XX/REART
0025      HH=HH/REART
0026      TD=285.00+T/24.00
0027      ARG1=PI2*TD/365.2500-0.49000
0028      ARG2=2.00*PI2*TD/365.2500-0.86200
0029      THETA=0.0017500*(T/24.00)+25.100*DSIN(ARG1)-9.200*DSIN(ARG2)
0030      FACTOR=PI2/86400000.00
0031      THETA=PI2*(T/24.00)+THETA*FACTOR
0032      OMG=0.0017500/24.00+(25.100*DCOS(ARG1)-18.400*DCOS(ARG2))
2*(PI2/8766.00)
      QMG=PI2/24.00+FACTOR*OMG
0033      RETURN
0034      END
0035

```

```

0001      SUPRGUTINE REDPI(A)
C*****
C*****
C*****      REDUCES AN ANGLE A IN RADIANS TO THE
C*****      ( -PI, +PI ) INTERVAL
C*****
C*****
C*****
0002      IMPLICIT REAL*8(A-H,L-Z)
0003      PI=4.00*DATAN(1.00)
0004      PI2=PI*2.00
0005      COPY=A/PI2
0006      IA=COPY
0007      IF(A.GE.0.00) A=A-DFLOAT(IA)*PI2
0008      IF(A.LT.0.00) A=A+DFLOAT(IA)*PI2
0009      5  IF(A.GT.-PI.AND.A.LE.PI) GO TO 8
0010      IF(A.LT.0.00) A=A+PI2
0011      IF(A.GT.0.00) A=A-PI2
0012      GO TO 5
0013      8  RETURN
0014      END

```

ORIGINAL PAGE IS
OF POOR QUALITY

2. Name of Program: PROGØ1

Function:

Adjustment of a set of observations with total time span equal to both earth and station step. Earth rotation parameters and station coordinates are treated as constants over the total time span of observations. A number of weighting options for radio source coordinates are available.

Input:

Number of weighting options for radio source coordinates, standard deviations of observations and radio source coordinates, approximate values of parameters, identification numbers of participating stations and radio source, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

REDPI.

(Following is a listing of PROGØ1.)

[illegible]

ORIGINAL PAGE IS
OF POOR QUALITY

0001
0002

```

      C      3,B(KK),NKEEP(K8,K8),SDQUA(10),PRATIO(10)
      C      WHERE: N=NO OF STATIONS, M=NO OF QUASARS, K=6+3*N+2*M, KK=K+2*N, K8=KK+8
      C
      C      READ EXPERIMENT IDENTIFICATION NO
0003      READ(5,100) IEXP
0004      WRITE(6,977) IEXP
0005      977      FORMAT('1'////10X,'EXPERIMENT NO =' ,I5/10X
      C      2,'*****',////)
      C
      C      READ NUMBER OF WEIGHTING OPTIONS
0006      READ(5,100) IOPT
0007      WRITE(6,978) IOPT
0008      978      FORMAT(10X,'REQUESTED NO OF WEIGHTING OPTIONS =' ,I5////)
      C
      C      READ A PRIORI STANDARD DEVIATIONS:
      C      SDBD : OF DELAY DISTANCE IN CM
      C      SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
      C      SDQUA: OF QUASAR COORDINATES IN CM
      C
0009      READ(5,101) SDBD,SDBF
0010      101      FORMAT(5F15.5)
      C
0011      DO 181 I=1,IOPT
0012      READ(5,101) SDQUA(I)
0013      PRATIO(I)=SDBD**2/SDQUA(I)**2
0014      181      CONTINUE
      C
      C
0015      R=6371000.D0
0016      CLIGHT=3.D0*(10.D0**8)
0017      CLIGHT=CLIGHT/R
0018      PI=4.D0*DATAN(1.D0)
0019      R100=R*100.D0
0020      RHRDM9=(12.D0/PI)*(10.D0**9)
0021      RASM3=(180.D0/PI)*3600000.D0
      C
0022      PF=(SDBD/R100)/(SDBF/R)
0023      PF=PF**2
      C      WEIGHTS :
      C      PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
      C
      C      READ IN= NO OF STATIONS & IM= NO OF QUASARS
0024      READ(5,100) IN, IM
0025      100      FORMAT(5I5)
0026      WRITE(6,920) IN,IM
0027      920      FORMAT(5X,'NO OF STATIONS =' ,I5/5X,'NO OF QUASARS =' ,I5)
0028      K=6+3*IN+2*IM
0029      KK=K+2*IN
0030      K8=KK+8
      C
      C      INITIALIZE POLAR MOTION (XP,HP) AND PRECESSION-NUTATION (XN,HN)
      C      PARAMETERS (APPROXIMATE VALUES) TO ZERO
0031      XP=0.D0
0032      HP=0.D0
0033      XN=0.D0
0034      HN=0.D0
      C
0035      DO 51 I=1,IN

```

```

0036      DO(I)=0.00
0037      DO(J)=0.00
0038      51  CONTINUE
C
C*****
C      READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, DMG,
C      X(I), Y(I), Z(I), (I=1,IN), RA(J), D(J), (J=1,IM)
C      ANGLES ARE IN DEGREES, DISTANCES IN METERS AND DMG IN RADIANS PER HOUR
C*****
C
0039      READ(5,101) TH
0040      WRITE(6,660) TH
0041      860  FORMAT(/5X,'THETA ZERO = ',F20.9/)
0042      TH=TH*PI/180.00
0043      GAM=PI/12.00
C
0044      WRITE(6,450)
0045      450  FORMAT(////,10X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
?,//3X,'STATIONS',10X,'LONG'
2,12X,'LAT',11X,'R',/)
C
0046      DO 5 I=1,IN
0047      READ(5,701) IDM,IL1,IL2,ALONG,APHI,AR
0048      WRITE(6,701) I,IL1,IL2,ALONG,APHI,AR
0049      701  FORMAT(1X,I2,1X,A4,A2,3F15.1)
0050      A PHI=APHI*PI/180.00
0051      ALONG=ALONG*PI/180.00
C      CONVERT TO CARTESIAN
0052      X(I)=AR*DCOS(APHI)*DCOS(ALONG)
0053      Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
0054      Z(I)=AR*DSIN(APHI)
C      CONVERT TO EARTH RADII UNITS
0055      X(I)=X(I)/R
0056      Y(I)=Y(I)/R
0057      Z(I)=Z(I)/R
0058      5    CONTINUE
C
C
0059      WRITE(6,451)
0060      451  FORMAT(/11X,'RADIO SOURCE',10X,'RA',17X,'DEC',/)
C
0061      DO 6 J=1,IM
0062      READ(5,321) IDM,IL1,IL2,IL3,RA(J),D(J)
0063      WRITE(6,321) J,IL1,IL2,IL3,RA(J),D(J)
0064      321  FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
C      CONVERT TO RADIAN UNITS
0065      RA(J)=RA(J)*PI/180.00
0066      D(J)=D(J)*PI/180.00
0067      6    CONTINUE
C
C*****
C      INITIATE N, U, XX, CC TO ZERO
C*****
C
0068      DO 7 I=1,KK
0069      XX(I)=0.00
0070      U(I)=0.00
0071      DO 7 J=1,8

```

ORIGINAL PAGE IS
OF POOR QUALITY

ORIGINAL PAGE IS
OF POOR QUALITY


```

0072      CC(J,I)=0.D0
0073      7      CONTINUE
          C
0074      DO 16 I=1,K8
0075      DO 16 J=1,K8
0076      N(I,J)=0.D0
0077      NKEEP(I,J)=0.D0
0078      16      CONTINUE
          C
          C      FORMULATE CONSTRAINT MATRIX CC
          C
0079      CC(5,1)=1.D0
0080      CC(4,2)=1.D0
0081      CC(6,5)=-1.D0
          C
0082      DO 3 I=1,IN
0083      IB1=4+3*I
0084      IB2=IB1+1
0085      IB3=IB2+1
0086      CC(1,IB1)=1.D0
0087      CC(2,IB2)=1.D0
0088      CC(3,IB3)=1.D0
0089      CC(4,IB2)=-Z(I)
0090      CC(4,IB3)=Y(I)
0091      CC(5,IB1)=Z(I)
0092      CC(5,IB3)=-X(I)
0093      CC(6,IB1)=-Y(I)
0094      CC(6,IB2)=X(I)
0095      IC1=K+I
0096      IC2=IC1+IN
0097      CC(7,IC1)=1.D0
0098      CC(8,IC2)=1.D0
0099      3      CONTINUE
          C
          C*****
          C      READ OBSERVATIONS AND IDENTIFIERS:
          C      DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
          C      AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
          C
          C*****
          C
0100      ICOUNT=0
          C
0101      99  READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
0102      104  FORMAT(3I5,3F15.5,2I5)
          C
          C      ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
          C      IF(ICHEK.EQ.1) GO TO 66
0103      IF(ICHEK.EQ.1) GO TO 66
0104      ICOUNT=ICOUNT+1
0105      WRITE(8,100) I,J,IP
          C
0106      96  DS=DS/R
0107      FRNG=FRNG/R
0108      DX=X(J)-X(I)
0109      DY=Y(J)-Y(I)
0110      DZ=Z(J)-Z(I)

```

C-2

FORTRAN IV G1 RELEASE 2.0

MAIN

DATE = 76300

21/55/22

```

0111      CA=DCOS(RA(IP))
0112      SA=DSIN(RA(IP))
0113      CD=DCOS(D(IP))
0114      SD=DSIN(D(IP))
0115      Y1=OMG*TK+TH
0116      Y2=Y1-RA(IP)
0117      CK=DCOS(Y1)
0118      SK=DSIN(Y1)
0119      CKP=DCOS(Y2)
0120      SKP=DSIN(Y2)

```

C
C
C

FORMULATE PARTIALS

```

0121      F1=-CD*DZ*CKP+DX*SD
0122      F1=F1+(-DX*XP+LY*HP)*CD*CKP-DZ*XP*SD
0123      F1=F1+DX*CD*(XN*CA-HN*SA)+DZ*SD*(XN*CK-HN*SK)
0124      F2=-CD*DZ*SKP-DY*SD
0125      F2=F2+DY*CD*(XP*CKP+HP*SKP)-DZ*HP*SD
0126      F2=F2+DY*CD*(-XN*CA+HN*SA)+DZ*SD*(XN*SK+HN*CK)
0127      F3=DX*(CD*CK*(-XN*CA+HN*SA)-SD*CK+XP*CD*CA)
2         +DY*(-CD*SK*(-XN*CA+HN*SA)+SD*SK-HP*CD*CA)
3         +DZ*(SD*(XP*CK+HP*SK-XN)+CD*CA)
0128      F4=DX*(CD*SA*(XN*CK-HN*SK-XP)+SK*SD)
2         +DY*(CD*SA*(-XN*SK-HN*CK+HP)+CK*SD)
3         +DZ*(SD*(-XP*SK+HP*CK-HN)-CD*SA)
0129      F5=-{DX*SKP+DY*CKP}*CD
0130      F5=F5+DZ*CD*(XP*SKP-HP*CKP)
0131      F5=F5+DX*SD*(XN*SK+HN*CK)+DY*SD*(XN*CK-HN*SK)
0132      F6= TK*F5
0133      F7=CD*CKP
0134      F7=F7+XP*SD
0135      F7=F7+SD*(-XN*CK+HN*SK)
0136      F8=-CD*SKP
0137      F8=F8-HP*SD
0138      F8=F8+SD*(XN*SK+HN*CK)
0139      F9=SD
0140      F9=F9+CD*(-XP*CKP-HP*SKP)
0141      F9=F9+CD*(XN*CA-HN*SA)
0142      F10=CD*(DX*SKP+DY*CKP)
0143      F10=F10+DZ*CD*(-XP*SKP+HP*CKP)
0144      F10=F10+DZ*CD*(-XN*SA-HN*CA)
0145      F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
0146      F11=F11+CD*(DX*XP-DY*HP)+DZ*SD*(XP*CKP+HP*SKP)
0147      F11=F11+DX*CD*(-XN*CK+HN*SK)+DY*CD*(XN*SK+HN*CK)
2         -DZ*SD*(XN*CA-HN*SA)
0148      F12=1.D0
0149      F12=TK/24.D0

```

C

```

0150      G1=DZ*CD*SKP
0151      G1=G1+DX*XP*CD*SKP-DY*HP*CD*SKP
0152      G1=G1-DZ*SD*(XN*SK+HN*CK)
0153      G1=OMG*C1
0154      G2=-DZ*CD*CKP
0155      G2=G2+DY*CD*(-XP*SKP+HP*CKP)
0156      G2=G2+DZ*SD*(XN*CK-HN*SK)
0157      G2=OMG*G2
0158      G3=(DX*SK+DY*CK)*SD
0159      G3=G3-(DX*SK+DY*CK)*CD*(-XN*CA+HN*SA)

```

ORIGINAL PAGE IS
OF POOR QUALITY

```

0160      G3=G3+DZ*SD*(-XP*SK+HP*CK)
0161      G3=DMG*G3
0162      G4=DX*CK*SD-DY*SK*SD
0163      G4=G4+DZ*SD*(-XP*CK+HP*SK)
0164      G4=G4+DX*CD*SA*(-XN*SK-HN*CK)+DY*CD*SA*(-XN*CK+HN*SK)
0165      G4=DMG*G4
0166      G5=-DX*CD*CKP+DY*CD*SKP
0167      G5=G5+DZ*CD*(XP*CKP+HP*SKP)
0168      G5=G5+DX*SD*(XN*CK-HN*SK)-DY*SD*(XN*SK+HN*CK)
0169      G5=DMG*G5
0170      G6=TK*G5
0171      G7=-CD*SKP
0172      G7=G7+SD*(XN*SK+HN*CK)
0173      G7=DMG*G7
0174      G8=-CD*CKP
0175      G8=G8+SD*(XN*CK-HN*SK)
0176      G8=DMG*G8
0177      G9=0.D0
0178      G9=G9+CD*(XP*SKP+HP*CKP)
0179      G9=DMG*G9
0180      G10=DX*CD*CKP-DY*CD*SKP
0181      G10=G10-DZ*CD*(XP*CKP+HP*SKP)
0182      G10=DMG*G10
0183      G11=DX*SD*SKP+DY*SD*CKP
0184      G11=G11+DZ*SD*(-XP*SKP+HP*CKP)
0185      G11=G11+DX*CD*(XN*SK+HN*CK)+DY*CD*(XN*CK-HN*SK)
0186      G11=DMG*G11
0187      G12=0.D0
0188      G13=1.D0/24.D0

```

C
C
C
C
C
C

A, B ARE ROWS OF THE DESIGN MATRIX CORRESPONDING TO DELAY
AND DELAY DERIVATIVE OBSERVATIONS RESPECTIVELY

INITIATE A AND B TO ZERO

```

0189      DO 8 II=1.KK
0190      A{II}=0.D0
0191      B{II}=0.D0
0192      CONTINUE

```

8
C
C
C

SET PARTIALS IN APPROPRIATE LOCATIONS OF A AND B ROWS

```

0193      A(1)=F1
0194      A(2)=F2
0195      A(3)=F3
0196      A(4)=F4
0197      A(5)=F5
0198      A(6)=F6

```

C

```

0199      B(1)=G1
0200      B(2)=G2
0201      B(3)=G3
0202      B(4)=G4
0203      B(5)=G5
0204      B(6)=G6

```

C

```

0205      I1=3*I+4
0206      I2=I1+1

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0207      I3=I1+2
0208      A(I1)=-F7
0209      A(I2)=-F8
0210      A(I3)=-F9
      C
0211      B(I1)=-G7
0212      B(I2)=-G8
0213      B(I3)=-G9
0214      IC1=K+1
0215      IC2=IC1+IN
0216      A(IC1)=-F12
0217      A(IC2)=-F13
      C
0218      B(IC1)=-G12
0219      B(IC2)=-G13
      C
0220      J1=3*J+4
0221      J2=J1+1
0222      J3=J1+2
0223      A(J1)= F7
0224      A(J2)= F8
0225      A(J3)= F9
      C
0226      B(J1)=G7
0227      B(J2)=G8
0228      B(J3)=G9
      C
0229      JC1=K+J
0230      JC2=JC1+IN
0231      A(JC1)=F12
0232      A(JC2)=F13
      C
0233      B(JC1)=G12
0234      B(JC2)=G13
      C
0235      IP1=3*IN+2*IP+5
0236      IP2=IP1+1
0237      A(IP1)=F10
0238      A(IP2)=F11
      C
0239      B(IP1)=G10
0240      B(IP2)=G11
      C
      C FORMULATION OF L VECTOR ENTRY
      C
0241      50 DS0=DX*F7+DY*F8+DZ*F9
0242      DS0=DS0+D0(J)-D0(I)+(D0(J)-D0(I))*(TK/24.D0)
0243      L=DS0-DS
```

```
      C
      C *****
      C
      C STORE PARTIALS / NON-ZERO ELEMENTS OF A & B ROWS ON DISK 8 FOR
      C LATER USE IN COMPUTATION OF RESIDUALS AND VTPV
      C ADD CONTRIBUTIONS OF CURRENT OBSERVATIONS TO NORMAL EQUATIONS
      C
      C *****
      C
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0244      WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0245      351  FORMAT(10X,F40.15)
          C
0246      DO 12 II=1,KK
0247      U(II)=U(II)-A(II)*L
0248      DO 12 JJ=1,KK
0249      NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
0250      12   CONTINUE
          C
          C
0251      FRNGO=DX*G7+DY*G8+DZ*G9
0252      FRNGO=FRNGO+(DD(J)-DD(I))/24.DO
0253      L=FRNGO-FRNG
          C
0254      WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
          C
0255      DO 65 II=1,KK
0256      U(II)=U(II)-B(II)*L*PF
0257      DO 65 JJ=1,KK
0258      NKEEP(II,JJ)=NKEEP(II,JJ)+B(II)*B(JJ)*PF
0259      65   CONTINUE
          C
0260      350  FORMAT(80X,2F19.9)
          C
0261      GO TO 33
          C
          C*****
          C
          C      SOLVE NORMAL EQUATIONS, COMPUTE PARAMETER CORRECTIONS
          C
          C*****
          C
          C
0262      66   DO 1 I=1,8
0263      DO 1 J=1,KK
0264      IK=I+KK
0265      NKEEP(IK,J)=CC(I,J)
0266      NKEEP(J,IK)=NKEEP(IK,J)
0267      1    CONTINUE
          C
0268      DO 182 IW=1,IOPT
          C
0269      DO 183 I=1,KP
0270      DO 183 J=1,K8
0271      N(I,J)=NKEEP(I,J)
0272      183  CONTINUE
          C
0273      IN1=7+IN*3
0274      DO 2 I=IN1,K
0275      N(I,I)=N(I,I)+PRATIO(IW)
0276      2    CONTINUE
          C
0277      CALL DMINV(N,K8,DET,L1,L2)
          C
0278      DO 187 I=1,KK
0279      XX(I)=0.DO
0280      187  CONTINUE
          C
```

```
0281      DO 14 I=1, KK
0282      DO 14 J=1, KK
0283      XX(I)=XX(I)+N(I,J)*U(J)
0284      14 CONTINUE
      C
      C      WRITE      XX
      C
0285      DO 9 I=1, 5
0286      CALL REDPI(XX(I))
0287      9 CONTINUE
      C
0288      ISTART=7+3*IN
      C
0289      DO 26 I=ISTART, K
0290      CALL REDPI(XX(I))
0291      26 CONTINUE
      C
      C
      C*****
      C      READ A AND B (ROWS OF DESIGN MATRIX) FROM DISK 8,
      C      FORMULATE RESIDUALS AND COMPUTE VARIANCE COVARIANCE MATRIX
      C*****
      C
0292      85 REWIND 8
0293      VTPV=0.0D0
      C
0294      VABS1=0.0D0
0295      VABS2=0.0D0
      C
0296      DO 22 II=1, ICOUNT
0297      READ(8,100) I, J, IP
      C
0298      DO 27 IDUM=1, 2
0299      READ(8,351) (F(JJ), JJ=1, 13), L
0300      V=0.0D0
      C
0301      DO 23 JJ=1, 6
0302      V=V+F(JJ)*XX(JJ)
0303      23 CONTINUE
      C
0304      I1=3*I+4
0305      I2=I1+1
0306      I3=I1+2
0307      J1=3*J+4
0308      J2=J1+1
0309      J3=J1+2
0310      V=V+F(7)*(XX(J1)-XX(I1))+F(8)*(XX(J2)-XX(I2))
      2      +F(9)*(XX(J3)-XX(I3))
      C
0311      IP1=3*IN+2*IP+5
0312      IP2=IP1+1
0313      V=V+F(10)*XX(IP1)+F(11)*XX(IP2)
      C
0314      IC1=K+I
0315      IC2=IC1+IN
0316      JC1=K+J
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0317      JC2=JC1+IN
0318      V=V+F(12)*(XX(JC1)-XX(IC1))+F(13)*(XX(JC2)-XX(IC2))
      C
0319      V=V+L
      C
0320      IF(IDUM.EQ.2) GO TO 28
0321      VABS1=VABS1+DABS(V)
0322      VTPV=VTPV+V**2
0323      GO TO 27
      C
0324      28 VABS2=VABS2+DABS(V)
0325      VTPV=VTPV+PF*V**2
0326      27 CONTINUE
      C
0327      22 CONTINUE
      C
0328      VABS1=VABS1/ICOUNT
0329      VABS2=VABS2/ICOUNT
0330      IIN=3*IN+6
0331      IIN1=IIN+1
      C
      C      ALTERNATIVE FOR SO COMPUTATION
      C      S0=VTPV+DFLOAT(IM*2)*(SD80/R100)**2
      C      S0=S0/DFLOAT(ICOUNT*2-KK+2*IM)
      C
0332      DO 172 I=IIN1,K
0333      VTPV=VTPV+PRATIO(IW)*XX(I)**2
0334      172 CONTINUE
0335      S0=VTPV/DFLOAT(2*ICOUNT-KK+2*IM)
      C
0336      DO 24 I=1,KK
0337      DO 24 J=1,KK
0338      N(I,J)=N(I,J)*S0
0339      24 CONTINUE
      C
      C
0340      K1=K+1
0341      KIN=K+IN
0342      KIN1=KIN+1
      C
0343      DO 41 J=1,KK
0344      N(1,J)=R100*N(1,J)
0345      N(2,J)=R100*N(2,J)
0346      N(3,J)=R100*N(3,J)
0347      N(4,J)=R100*N(4,J)
0348      N(5,J)=RASM3*N(5,J)
0349      N(6,J)=RHRD49*N(6,J)
      C
0350      DO 42 I=7,IIN
0351      N(I,J)=N(I,J)*R100
0352      42 CONTINUE
      C
0353      DO 43 I=IIN1,K
0354      N(I,J)=N(I,J)*RASM3
0355      43 CONTINUE
0356      DO 47 I=K1,KIN
0357      N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0358      47 CONTINUE
```

ORIGINAL PAGE IS
OF POOR QUALITY

```

      C
0359      DO 48 I=KINI, KK
0360      N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0361      CONTINUE
      C
0362      41 CONTINUE
      C
0363      DO 44 I=1, KK
0364      N(I,1)=R100*N(I,1)
0365      N(I,2)=R100*N(I,2)
0366      N(I,3)=R100*N(I,3)
0367      N(I,4)=R100*N(I,4)
0368      N(I,5)=RASM3*N(I,5)
0369      N(I,6)=RHRDM9*N(I,6)
      C
0370      DO 45 J=7, IIN
0371      N(I,J)=N(I,J)*R100
0372      CONTINUE
      C
0373      DO 46 J=IINI, K
0374      N(I,J)=N(I,J)*RASM3
0375      CONTINUE
      C
0376      DO 49 J=K1, KIN
0377      N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0378      CONTINUE
      C
0379      DO 52 J=KINI, KK
0380      N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0381      CONTINUE
      C
0382      44 CONTINUE
      C
0383      227 FORMAT(10X,3F20.4)
0384      229 FORMAT(10X,2F22.6)
0385      222 FORMAT(5X,F40.9)
      C
0386      DO 86 I=1, KK
0387      N(I,1)=DSQRT(N(I,1))
0388      CONTINUE
      C
0389      ICNT2=ICOUNT*2
0390      IDF=ICNT2-KK+2*IM
0391      WRITE(6,696) ICNT2, IDF
0392      696 FORMAT(////5X, 'NO OF OBSERVATIONS =', I5//5X
      2, 'DEGREES OF FREEDOM =', I5/)
      C
0393      WRITE(6,850) IEXP, SDBD, SDBF, SDQUA(IW)
0394      850 FORMAT('1'////15X, 'EXPERIMENT NO =', I5///10X
      1, 'A PRIORI STANDARD DEVIATIONS', //5X
      2, 'DELAY DISTANCES (CM)', 10X, F15.5//5X
      3, 'DISTANCE DERIVATIVES (M/HOUR)', 1X, F15.5//5X
      4, 'QUASAR COORDINATES (CM)', 7X, F15.5/)
      C
0395      WRITE(6,356) (N(I,I), I=1,6)
0396      356 FORMAT(///20X, 'STANDARD DEVIATIONS'/20X
      1, '*****'//5X
      2, 'POLAR MOTION =', 7X, F14.5, 3X, 'CM', /26X, F14.5, 3X, 'CM'/5X

```



```
      3, 'PRECESSION-NUTATION =', F14.5, 3X, 'CM', /26X, F14.5, 3X, 'CM', /5X
      4, 'THETA ZERO =', 9X, F14.5, 3X, 'ARCSEC/1000' /5X
      5, 'OMG =', 16X, F14.5, 3X, 'REV PER DAY/10**9' //10X
      6, 'STATION COORDINATES (X,Y,Z) IN CM'//
0397      WRITE(6,357) (N(I,I), I=7, IIN)
0398      357  FORMAT(10X, 3F20.5)
0399      WRITE(6,358)
0400      358  FORMAT(/10X, 'QUASAR COORDINATES (RA, DEC) IN ARCSEC/1000'//)
0401      WRITE(6,359) (N(I,I), I=IIN1, K)
0402      359  FORMAT(10X, 2F20.5)
      C
0403      WRITE(6,361)
0404      361  FORMAT(/10X, 'STATION CLOCK OFFSETS IN MSEC'//)
0405      WRITE(6,362) (N(I,I), I=K1, KIN)
0406      362  FORMAT(2X, F20.9)
0407      WRITE(6,363)
0408      363  FORMAT(/10X, 'STATION CLOCK DRIFTS IN MSEC PER DAY'//)
0409      WRITE(6,362) (N(I,I), I=KIN1, KK)
      C
      C
0410      162  CONTINUE
      C
0411      86   STOP
0412      END
```

3. Name of Program: PROGØ2

Function:

Adjustment of observations with a time span equal to the station step.

Station coordinates are considered constant over the whole set of observations, while earth rotation parameters are treated as constant but different over a prearranged set of subintervals.

Input:

Standard deviations of observations and radio source coordinates, approximate values of parameters, station and radio source identification numbers, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

MADD, MATPRO, TRAPRO, PROTRA for matrix operations and REDPI.

(Following is a listing of PROGØ2 and supporting subroutines, except for REDPI already included in VLBI SIMULATOR.)

```
C*****  
C*****  
C*****          PROG02  
C*****  
C*****          VLRI ADJUSTMENT PROGRAM  
C*****  
C*****          THIS VERSION PERFORMS THE ADJUSTMENT OF  
C*****          A SET OF OBSERVATIONS TREATING STATION  
C*****          COORDINATES AS CONSTANTS. EARTH ROTATION  
C*****          PARAMFTERS ARE CONSTANT BUT DIFFERENT  
C*****          OVER SUBINTERVALS OF THE ORIGINAL TOTAL  
C*****          TIME INTERVAL OF THE AVAILABLE OBSERVATIONS.  
C*****  
C*****          OBSERVATIONS: DELAY DISTANCES AND  
C*****                   THEIR DERIVATIVES  
C*****  
C*****          UNKNOWNNS: EARTH MOTIION PARAMETERS  
C*****                   STATION & QUASAR COORDINATES  
C*****                   CLOCK OFFSETS AND DRIFTS  
C*****  
C*****          SYSTEMS DEFINITION:  
C*****          QUASAR FIXED: WEIGHTED QUASAR COORDINATES  
C*****          EARTH FIXED: BY MEANS OF INNER CONSTRAINTS  
C*****  
C*****          SUBROUTINES REQUIRED :  
C*****                REDPI  
C*****                MADD  
C*****                MATPRO  
C*****                TRAPRO  
C*****                PROTRA
```

```

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION DO(3),DD(3),X(3),Y(3),Z(3),RA(9),D(9),ED(33,8),U(39)
2,NKEEP(39,39),A(39),B(39),NB(6,33),UDI(33),HI(6,33),RI(33,33)
3,QF(41,41),RLI(33),H(6,33),RL(33),UD(33),HTED(33,8),L1K(41)
4,L2K(41),GB(33,33),FB(33,8),GBL(33),FET(33),GU(33),FBEO(33,6)
5,GBHT(33,6),FETN(33,6),XBI(6,20),STD(153),XB(33)
6,FDO(6,8),UDDI(6),NDD(6,6),L16(6),L26(6),TI(6),RK(6,6),RT(6)
7,RKED(6,8),ETKE(8,8),FF(8,8),ETOT(8),FEDT(8),FBTL(8),FBTU(8)
8,SQL(6),CDF(6,8),EFET(6,6),GDD(6,6),HGHT(6,6),HFEN(6,6),EFEN(6,6)
9,NFFEN(6,6),HIXB(6),NSQL(6),F(13)
C DIMENSION DO(IN),DD(IN),X(IN),Y(IN),Z(IN),RA(IM,D(IM),ED(K,8)
C 2,U(KK),NKEEP(KK,KK),A(KK),B(KK),NB(6,K)UDI(K),HI(6,K),RI(K,K)
C 3,QB(K8,K8),RLI(K),H(K),RL(K),UD(K),HTED(K,8),L1K(K),L2K(K)
C 4,GP(K,K),FB(K,8),GBL(K),FET(K),GU(K),FBEO(K,6),GBHT(K,6)
C 5,FETN(K,6),XBI(6,IDCNT),STD(6*IDCNT+K),XB(K),
C 6,FDO(6,8),UDDI(6),VDD(6,6),L16(6),L26(6),TI(6),RK(6,6),RT(6)
C 7,RKFDO(6,8),ETKE(8,8),FF(8,8),ETOT(9),FEDT(8),FBTL(8),FBTU(8)
C 8,SQL(6),CDF(6,8),EFET(6,6),GDD(6,6),HGHT(6,6),HFEN(6,6),EFEN(6,6)
C 9,NEFEN(6,6)HIXB(6),NSQL(6),F(13)
C WHERE: K=5*IN+2*IM, KK=K+6, KB=K+8, IN=NO OF STATIONS, IM=NO OF QUASARS
EQUIVALENCF {HTED,FB},{UDI,L1K,GBL},{RLI,L2K,FET},{RI,GB}
2,{RKED,EDF,GBHT},{EDO,FETN},{RT,HIXB},{L16,TI},{L26,NSQL}
3,{RK,NEFEN},{HGHT,HFEN,EFEN},{ETKE,FF},{ETOT,FBTL,FBTU}

```

0001
0002

ORIGINAL PAGE IS
OF POOR QUALITY

0003

```

      C      READ EXPERIMENT IDENTIFICATION NO
0004      READ(5,100) IEXP
0005      WRITE(6,977) IEXP
0006      977  FORMAT('1'////10X,'EXPERIMENT NO =' ,I5/10X
           2,'*****'////)
      C
      C
      C      READ A PRIORI STANDARD DEVIATIONS:
      C      SDBD : OF DELAY DISTANCE IN CM
      C      SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
      C      SDQUA: OF QUASAR COORDINATES IN CM
      C
0007      READ(5,101) SDBD,SDBF
0008      101  FORMAT(5F15.5)
      C
0009      WRITE(6,240) SDBD,SDBF
0010      240  FORMAT(10X,'OBSERVATIONAL NOISE'/15X,'DISTANCES
           2,'F10.5/15X,'DISTANCE RATES ',F10.5//)
      C
0011      READ(5,101) SDQUA
0012      PRATIO=SDBD**2/SDQUA**2
      C
      C      READ EARTH STEP INTERVAL IN HOURS
      C
0013      READ(5,101) FRSTEP
0014      WRITE(6,243) ERSTEP
0015      243  FORMAT(/10X,'EARTH STEP INTERVAL (HRS) =' ,F10.2/)
      C
0016      R=6371000.00
0017      CLIGHT=3.00*(10.00**8)
0018      CLIGHT=CLIGHT/R
0019      PI=4.00*ATAN(1.00)
0020      R100=R*100.00
0021      KHKDM9=(12.00/PI)*(10.00**9)
0022      RASM3=(180.00/PI)*3600000.00
      C
0023      PF=(SDBD/R100)/(SDBF/R)
0024      PF=PF**2
      C      PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
      C
      C      READ IN= NO OF STATIONS & IM= NO OF QUASARS
      C*****
0025      READ(5,100) IN, IM
0026      100  FORMAT(10I5)
0027      WRITE(6,920) IN,IM
0028      920  FORMAT(5X,'NO OF STATIONS =' ,I5/5X,'NO OF QUASARS  =' ,I5)
0029      K=5*IN+2*IM
0030      KK=K+6
0031      KR=K+8
      C
      C
0032      DO 51 I=1,IN
0033      DD(I)=0.00
0034      DD(I)=0.00
0035      51  CONTINUE
      C
      C      READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, DMG,
      C*****

```

```

      C      X(I), Y(I), Z(I), (I=1,IN), RA(J), D(J), (J=1,IM)
      C      ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
0036      READ(5,101) TH
0037      WRITE(6,860) TH
0038      860  FORMAT(/5X,'THETA ZERO = ',F20.9/)
0039      THET=TH*PI/180.00
0040      OMG=PI/12.00

      C
0041      WRITE(6,450)
0042      450  FORMAT(///,10X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
      2,/,/3X,'STATIONS',10X,'LONG'
      2,12X,'LAT',11X,'R',/)

      C
0043      DO 5 I=1,IN
0044      READ(5,701) IDM,IL1,IL2,ALONG,APHI,AR
0045      WRITE(6,701) I,IL1,IL2,ALONG,APHI,AR
0046      701  FORMAT(1X,I2,1X,A4,A2,3F15.1)
0047      APHI=APHI*PI/180.00
0048      ALONG=ALONG*PI/180.00
      C      CONVERT TO CARTESIAN
0049      X(I)=AR*DCOS(APHI)*DCOS(ALONG)
0050      Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
0051      Z(I)=AR*DSIN(APHI)
      C      CONVERT TO EARTH RADII UNITS
0052      X(I)=X(I)/R
0053      Y(I)=Y(I)/R
0054      Z(I)=Z(I)/R
0055      5  CONTINUE
      C
      C
0056      WRITE(6,451)
0057      451  FORMAT(/11X,'RADIO SOURCE',10X,'RA',17X,'DEC',/)

      C
0058      DO 6 J=1,IM
0059      READ(5,321) IDM,IL1,IL2,IL3,RA(J),D(J)
0060      WRITE(6,321) J,IL1,IL2,IL3,RA(J),D(J)
0061      321  FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
      C      CONVERT TO RADIAN UNITS
0062      RA(J)=RA(J)*PI/180.00
0063      D(J) = D(J)*PI/180.00
0064      6  CONTINUE
      C
      C
      C      FORMULATE CONSTRAINT MATRICES ED, EDO
0065      DO 11 J=1,8
0066      DO 9 I=1,6
0067      ED(I,J)=0.00
0068      9  CONTINUE
0069      DO 11 I=1,K
0070      ED(I,J)=0.00
0071      11 CONTINUE
      C
0072      EDO(1,5)=1.00
0073      EDO(2,4)=1.00
0074      EDO(5,6)=-1.00
      C
0075      DO 3 I=1,IN
0076      IB1=3*I-2

```

```

0077      IE2=IB1+1
0078      IB3=IA2+1
0079      ED(IP1,1)=1.00
0080      ED(IE2,2)=1.00
0081      ED(IB3,3)=1.00
0082      ED(IE2,4)=-Z(I)
0083      ED(IB3,4)=Y(I)
0084      ED(IA1,5)=Z(I)
0085      ED(IP3,5)=-X(I)
0086      ED(IA1,6)=-Y(I)
0087      ED(IE2,6)=X(I)
0088      IC1=3*IN+2*IM+I
0089      IC2=IC1+IN
0090      ED(IC1,7)=1.00
0091      ED(IC2,8)=1.00
0092      3      CONTINUE
0093      IDCNT=0
0094      ICOUNT=0

C
C      INITIATE H, RK, RL, RT, UD, QB TO ZERO
C*****
C
0095      DO 71 I=1,K
0096      KL(I)=0.00
0097      UD(I)=0.00
0098      DO 71 J=1,6
0099      H(J,I)=0.00
0100      71      CONTINUE
C
0101      DO 72 I=1,6
0102      RT(I)=0.00
0103      DO 72 J=1,6
0104      RK(I,J)=0.00
0105      72      CONTINUE
0106      DO 73 I=1,K8
0107      DO 73 J=1,K8
0108      QB(I,J)=0.00
0109      73      CONTINUE
C
C
C      INITIATE NKEEP,U TO ZERO
C*****
0110      55      DO 16 I=1,KK
0111      U(I)=0.00
0112      DO 16 J=1,KK
0113      NKEEP(I,J)=0.00
0114      16      CONTINUE
C
0115      TH=THET+(EKSTEP/12.00)*PI*DFLOAT(IDCNT)
C
C*****
C
C      READ OBSERVATIONS AND IDENTIFIERS:
C      DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
C      AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
C*****
C
C

```

ORIGINAL PAGE IS
OF POOR QUALITY

```

0116      C
0117      33  READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
          104  FORMAT(3I5,3F15.5,2I5)
          C
          C      ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
0118      IF(ICHEK.EQ.1) GO TO 66
0119      IF(ICHEK.EQ.2) GO TO 66
0120      ICOUNT=ICOUNT+1
0121      TK=TK-FRSTEP*DFLOAT(ICOUNT)

          C
0122      DS=DS/R
0123      FRNG=FRNG/R
0124      DX=X(J)-X(I)
0125      DY=Y(J)-Y(I)
0126      DZ=Z(J)-Z(I)
0127      CA=DCOS(RA(IP))
0128      SA=DSIN(RA(IP))
0129      CD=DCOS(D(IP))
0130      SD=DSIN(D(IP))
0131      Y1=OMG+TK+TH
0132      Y2=Y1-RA(IP)
0133      CK=DCOS(Y1)
0134      SK=DSIN(Y1)
0135      CKP=DCOS(Y2)
0136      SKP=DSIN(Y2)

          C
          C      FORMULATE PARTIALS
          C
0137      F1=-CD*DZ*CKP+DX*SD
0138      F2=-CD*DZ*SKP-DY*SD
0139      F3=-DX*SD*CK+DY*SD*SK+DZ*CD*CA
0140      F4=DX*SK*SD+DY*CK*SD-DZ*CD*SA
0141      F5=- (DX*SKP+DY*CKP)*CD
0142      F6= TK*F5
0143      F7=CD*CKP
0144      F8=-CD*SKP
0145      F9=SD
0146      F10=CD*(DX*SKP+DY*CKP)
0147      F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
0148      F12=1.D0
0149      F13=TK/24.D0

          C
0150      G1=DZ*CD*SKP
0151      G1=OMG*G1
0152      G2=-DZ*CD*CKP
0153      G2=OMG*G2
0154      G3=(DX*SK+DY*CK)*SD
0155      G3=OMG*G3
0156      G4=DX*CK*SD-DY*SK*SD
0157      G4=OMG*G4
0158      G5=-DX*CD*CKP+DY*CD*SKP
0159      G5=OMG*G5
0160      G6=TK*G5
0161      G7=-CD*SKP
0162      G7=OMG*G7
0163      G8=-CD*CKP
0164      G8=OMG*G8
0165      G9=0.D0

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0166      G10=DX*CD*CKP-DY*CD*SKP
0167      G10=OMG*G10
0168      G11=DX*SD*SKP+DY*SD*CKP
0169      G11=OMG*G11
0170      G12=0.D0
0171      G13=1.D0/24.D0
```

```
C
C      INITIATE A AND B TO ZERO
```

```
C*****
```

```
0172      DO 8 II=1, KK
0173      A(II)=0.D0
0174      B(II)=0.D0
0175      8 CONTINUE
```

```
C
C      SET PARTIALS IN APPROPRIATE LOCATIONS OF A ROW
```

```
C*****
```

```
0176      A(1)=F1
0177      A(2)=F2
0178      A(3)=F3
0179      A(4)=F4
0180      A(5)=F5
0181      A(6)=F6
```

```
C
      B(1)=G1
      B(2)=G2
      B(3)=G3
      B(4)=G4
      B(5)=G5
      B(6)=G6
```

```
0188      I1=3*I+4
0189      I2=I1+1
0190      I3=I1+2
0191      A(I1)=-F7
0192      A(I2)=-F8
0193      A(I3)=-F9
```

```
C
      B(I1)=-G7
      B(I2)=-G8
      B(I3)=-G9
```

```
0197      IC1=6+3*IN+2*IM+I
0198      IC2=IC1+IN
0199      A(IC1)=-F12
0200      A(IC2)=-F13
```

```
0201      B(IC1)=-G12
0202      B(IC2)=-G13
```

```
0203      J1=3*J+4
0204      J2=J1+1
0205      J3=J1+2
0206      A(J1)= F7
0207      A(J2)= F8
0208      A(J3)= F9
```

```
C
```



```

0209      B(J1)=G7
0210      B(J2)=G8
0211      B(J3)=G9
      C
0212      JC1=6+3*IN+2*IM+J
0213      JC2=JC1+IN
0214      A(JC1)=F12
0215      A(JC2)=F13
      C
0216      B(JC1)=G12
0217      B(JC2)=G13
      C
0218      IP1=3*IN+2*IP+5
0219      IP2=IP1+1
0220      A(IP1)=F10
0221      A(IP2)=F11
      C
0222      B(IP1)=G10
0223      B(IP2)=G11
      C
      C      FORMULATION OF L VECTOR ENTRY
      C
0224      50      DS0=DX*F7+DY*F8+DZ*F9
0225      DS0=DS0+D0(J)-D0(I)+(DD(J)-DD(I))*(TK/24.D0)
0226      L=DS0-D5
      LW1=L*R100
      C
      C
      C      *****
      C      STORE A, B, L AND OBSERVATION IDENTIFIERS ON DISK 8 FOR LATER
      C      USE IN COMPUTATION OF RESIDUALS AND COVARIANCES
      C      *****
      C
0227      WRITE(8,100) ICHEK,I,J,IP
0228      WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0229      351      FORMAT(10X,F40.15)
      C
0230      DO 12 II=1,KK
0231      U(II)=U(II)+A(II)*L
0232      DO 12 JJ=1,KK
0233      NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
0234      12      CONTINUE
      C
      C
0235      FRNG0=DX*G7+DY*G8+DZ*G9
0236      FRNG0=FRNG0+(DD(J)-DD(I))/24.D0
0237      L=FRNG0-FRNG
      LW2=L*R
      C
      C      WRITE(6,350) LW1,LW2
      C
0238      WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
0239      DO 65 II=1,KK
0240      U(II)=U(II)+B(II)*L*PF
0241      DO 65 JJ=1,KK
0242      NKEEP(II,JJ)=NKEEP(II,JJ)+B(II)*B(JJ)*PF
0243      65      CONTINUE
      C
0244      350      FORMAT(60X,2F19.9)

```

???????

```

C
0245      GO TO 33
C
C
C*****
C
C
0246      66  WRITE(8,100) ICHEK
C
0247      DO 20 I=1,6
0248      UDDI(I)=U(I)
0249      DO 20 J=1,6
0250      NDD(I,J)=NKEEP(I,J)
0251      20  CONTINUE
0252      DO 21 I=1,6
0253      DO 21 J=1,K
0254      J6=J+6
0255      NB(I,J)=NKEEP(I,J6)
0256      21  CONTINUE
C
0257      DO 22 I=1,K
0258      I6=I+6
0259      UDI(I)=U(I6)
0260      22  CONTINUE
C
0261      CALL DMINV(NDD,6,DFT,L16,L26)
0262      CALL MATPRO(NDD,NB,HI,6,6,K)
0263      CALL TRAPRO(NB,HI,RI,K,6,K)
0264      DO 23 I=1,K
0265      DO 23 J=1,K
0266      I6=I+6
0267      J6=J+6
0268      QB(I,J)=QB(I,J)+NKEEP(I6,J6)-RI(I,J)
0269      23  CONTINUE
C
0270      CALL MATPRO(NDD,UDDI,TI,6,6,1)
0271      CALL TRAPRO(NB,TI,RLI,K,6,1)
0272      CALL MADD(H,HI,6,K)
0273      CALL MADD(RK,NDD,6,6)
0274      CALL MADD(RL,RLI,K,1)
0275      CALL MADD(RT,TI,6,1)
0276      CALL MADD(UD,UOI,K,1)
C
C*****
C      STORE UDDI,NDD,HI,TI ON DISK 9
C*****
C
0277      DO 25 I=1,6
0278      DO 25 J=1,6
0279      WRITE(9,351) NDD(I,J)
0280      25  CONTINUE
C
0281      DO 26 I=1,6
0282      DO 26 J=1,K
0283      WRITE(9,351) HI(I,J)
0284      26  CONTINUE
C
0285      DO 27 I=1,6

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0286      WRITE(9,351) TI(I)
0287      CONTINUE
0288      IDCNT=IDCNT+1
0289      IF(ICHEK.EQ.2) GO TO 67
0290      GO TO 55

C
C
C*****
C
C
0291      67      CALL TRAPRO(H,EDO,HTEDO,K,6,8)
C
0292      DO 28 I=1,K
0293      DO 28 J=1,8
0294      JK=J+K
0295      QB(I,JK)=ED(I,J)-HTEDO(I,J)
0296      QR(JK,I)=QR(I,JK)
0297      28      CONTINUE
C
0298      CALL MATPRO(RK,EDO,RKEDO,6,6,8)
0299      CALL TRAPRO(EDO,RKEDO,ETKE,8,6,8)
C
0300      DO 29 I=1,8
0301      IK=I+K
0302      DO 29 J=1,8
0303      JK=J+K
0304      QB(IK,JK)=-ETKE(I,J)
0305      29      CONTINUE
C
0306      IPF=3*IN+1
0307      IPL=3*IN+2*IM
0308      DO 34 I=IPF,IPL
0309      QB(I,I)=QB(I,I)+PRATIO
0310      34      CONTINUE
C
0311      CALL DMINV(QB,K8,DET,L1K,L2K)
C
0312      DO 30 I=1,K
0313      DO 30 J=1,K
0314      GR(I,J)=QB(I,J)
0315      30      CONTINUE
C
0316      DO 31 I=1,K
0317      DO 31 J=1,8
0318      JK=J+K
0319      FB(I,J)=QB(I,JK)
0320      31      CONTINUE
C
0321      DO 32 I=1,8
0322      IK=I+K
0323      DO 32 J=1,8
0324      JK=J+K
0325      FF(I,J)=QB(IK,JK)
0326      32      CONTINUE
C
0327      CALL MATPRO(GB,RL,GBL,K,K,1)
0328      CALL TRAPRO(EDO,RT,ETOT,8,6,1)
0329      CALL MATPRO(FB,ETOT,FET,K,8,1)
```

```

0330      CALL MATPRO(GB,UD,GU,K,K,1)
          C
0331      DO 41 I=1,K
0332      XB(I)=GBL(1)+FET(I)-GU(I)
0333      41 CONTINUE
          C
0334      CALL MATPRO(FF,ETOT,FEDT,8,8,1)
0335      CALL TRAPRO(FB,RL,F8TL,8,K,1)
0336      CALL MADD(FEDT,F8TL,8,1)
0337      CALL TRAPRO(FB,UD,F8TU,8,K,1)
          C
0338      DO 44 I=1,8
0339      FEDT(I)=FEDT(I)-F8TU(I)
0340      44 CONTINUE
          C
0341      CALL MATPRO(EDO,FEDT,SOL,6,8,1)
0342      CALL PROTRA(FB,EDO,FBEOT,K,8,6)
0343      CALL MATPRO(ED0,FF,EDF,6,8,8)
0344      CALL PROTRA(EDF,ED0,EFET,6,8,6)
          C
0345      REWIND 9
          C
0346      DO 60 IDAY=1,INDNT
          C
          C
0347      DO 35 I=1,6
0348      DO 35 J=1,6
0349      READ(9,351) NDD(I,J)
0350      35 CONTINUE
          C
0351      DO 36 I=1,6
0352      DO 36 J=1,K
0353      READ(9,351) HI(I,J)
0354      36 CONTINUE
          C
0355      DO 37 I=1,6
0356      READ(9,351) TI(I)
0357      37 CONTINUE
          C
0358      DO 38 I=1,6
0359      DO 38 J=1,6
0360      GDD(I,J)=NDD(I,J)
0361      38 CONTINUE
          C
0362      CALL PROTRA(GB,HI,GBHT,K,K,6)
0363      CALL MATPRO(HI,GBHT,HGHT,6,K,6)
0364      CALL MADD(GDD,HGHT,6,6)
0365      CALL MATPRO(FBEOT,NDD,FETN,K,6,6)
0366      CALL MATPRO(HI,FETN,HFEN,6,K,6)
          C
0367      DO 39 I=1,6
0368      DO 39 J=1,6
0369      GDD(I,J)=GDD(I,J)+HFEN(I,J)+HFEN(J,I)
0370      39 CONTINUE
0371      CALL MATPRO(EFET,NDD,EFEN,6,6,6)
0372      CALL MATPRO(NDD,EFEN,NEFEN,6,6,6)
0373      CALL MADD(GDD,NEFEN,6,6)
          C

```

ORIGINAL PAGE IS
OF POOR QUALITY

```

      C
      C
      C
0374      DO 48 I=1,6
0375      IPF=6*IDAY-6+I
0376      STD(IPF)=GDD(I,I)
0377      48 CONTINUE
      C
      C
0378      CALL MATPRO(NDD,SOL,NSOL,6,6,1)
0379      CALL MATPRO(HI,XB,HIXB,6,K,1)
      C
0380      DO 40 I=1,6
0381      XBI(I,IDAY)=-TI(I)-NSOL(I)-HIXB(I)
0382      40 CONTINUE
0383      60 CONTINUE
      C
      C
      C*****
      C
      C
0384      DO 49 I=1,K
0385      IPL=6*ICNT+I
0386      STD(IPL)=GB(I,I)
0387      49 CONTINUE
      C
0388      REWIND 8
0389      IDAY=1
0390      VTPV=0.00
      C
0391      77 READ(8,100) ICHEK,I,J,IP
0392      IF(ICHEK.FQ.1) GO TO 88
0393      IF(ICHEK.EQ.2) GO TO 89
0394      IS1=3*I-2
0395      IS2=IS1+1
0396      IS3=IS2+1
0397      JS1=3*J-2
0398      JS2=JS1+1
0399      JS3=JS2+1
0400      IP1=3*IN+2*IP-1
0401      IP2=IP1+1
0402      ID1=3*IN+2*IM+I
0403      JD1=3*IN+2*IM+J
0404      ID2=ID1+IN
0405      JD2=JD1+IN
0406      DO 45 JJ=1,2
0407      READ(8,351) (F(II),II=1,13),L
0408      V=L
0409      DO 42 II=1,6
0410      V=V+F(II)*XBI(II,IDAY)
0411      42 CONTINUE
      C
0412      V=V+F(7)*(XB(JS1)-XB(IS1))+F(8)*(XB(JS2)-XB(IS2))
      2 +F(9)*(XB(JS3)-XB(IS3))
0413      V=V+F(10)*XB(IP1)+F(11)*XB(IP2)
0414      V=V+F(12)*(XB(JD1)-XB(ID1))+F(13)*(XB(JD2)-XB(ID2))
0415      IF(JJ.EQ.2) GO TO 43
      C      VW=V*R100

```

```

0416      C      WRITE(6,453) VW
0417      VTPV =VTPV+V**2
0418      GO TO 45
      43      VTPV=VTPV+PF*V**2
      C      VW=V*R
      C      WRITE(6,454) VW
0454      FORMAT(50X,F30.15)
0453      FORMAT(20X,F30.15)
0419      45      CONTINUE
0420      GO TO 77
0421      86      IDAY=IDAY+1
0422      GO TO 77
0423      89      IPF=3*IN+1
0424      IPL=3*IN+2*IM
0425      DO 46      I=IPF,IPL
0426      VTPV=VTPV+PRATIO*X6(I)**2
0427      46      CONTINUE
      C
0428      IS2=2*ICOUNT
0429      JS2=5*IN+6*ICNT
0430      IS3=IS2-JS2
0431      IDC=(24.00/ERSTEP+0.1D0)
0432      IDC=IDC/IDC
0433      WRITE(6,242) IDC,IS2,JS2,IS3
0434      242      FORMAT(///10X,'DAYS OF OBSERVATION=',I5/10X,'NO OF OBSERVATIONS ='
0435      2,I5/10X,'NO OF UNKNOWN =',I5/10X,'DEGREES OF FREEDOM =',I5//)
      S0=VTPV/DFLOAT(IS3)
      C
0436      IPL=K+6*IDCNT
0437      DO 47 I=1,IPL
0438      IF(STD(I).GT.0.D0) GO TO 999
0439      WRITE(6,998) I,STD(I)
0440      998      FORMAT(///// '***** ERROR *****      I=',I5/10X'STD=',F90.30/////))
0441      999      STD(I)=DSQRT(STD(I)*S0)
0442      47      CONTINUE
      C
0443      C*****      WRITE STANDARD DEVIATIONS
0444      888      WRITE(6,888)
      888      FORMAT('1'///10X,'STANDARD DEVIATIONS'/10X,'*****'//
0445      2,3X,'DAY',10X,'POLAR MOTION',13X,'PRECESSION - NUTATION',7X
0446      3,'THETA ZERO',7X,'OMEGA'/12X,'KSI (CM)',7X,'ETA (CM)',7X
0447      4,'KSI (CM)',7X,'ETA (CM)',9X,'(CM)',6X,'(REV/DAY)*10**9'//)
      C
0448      DO 85 IDAY=1,IDCNT
0449      IPL=6*IDAY
0450      IPF=IPL-5
0451      DO 86 II=1,5
0452      JJ=IPF+II-1
0453      CALL REDPI(STD(JJ))
0454      STD(JJ)=STD(JJ)*R100
0455      86      CONTINUE
0456      STD(IPL)=STD(IPL)*RHRDM9
0457      WRITE(6,889) IDAY,(STD(J),J=IPF,IPL)
0458      889      FORMAT(/1X,I4,6F15.5)
0459      85      CONTINUE
0460      WRITE(6,221)
0461      221      FORMAT('1'///10X,'STATION COORDINATES',//,17X,'X',9X,'Y',9X
0462      2,'Z'/15X,'(CM)',6X,'(CM)',6X,'(CM)'//)

```

 ORIGINAL PAGE IS
OF POOR QUALITY

```
0459      IPF=6*IDCNT+1
0460      IPL=6*IDCNT+3*IN+2*IM
0461      II=IPL-2*IM+1
0462      DO 91 I=II,IPL
0463      CALL RFOPI(STD(I))
0464      91 CONTINUE
      C
0465      DO 75 I=IPF,IPL
0466      STD(I)=STD(I)*R100
0467      75 CONTINUE
      C
0468      IPL=IPL-2*IM
0469      WRITE(6,222) (STD(I),I=IPF,IPL)
0470      222 FORMAT(10X,3F10.2)
      C
0471      WRITE(6,223)
0472      223 FORMAT(////10X,'QUASAR COORDINATES',//16X,'RA',8X,'DEC',/15X
2,'(CM)',6X,'(CM)'//)
0473      IPF=IPL+1
0474      IPL=IPL+2*IM
0475      WRITE(6,224) (STD(I),I=IPF,IPL)
0476      224 FORMAT(10X,2F10.2)
      C
0477      WRITE(6,225)
0478      225 FORMAT(////10X,'STATION CLOCK OFFSETS AND DRIFTS'/13X
2,'(MSEC)',6X,'(MSEC/DAY)'//)
      C
0479      DO 76 I=1,IN
0480      IC1=IPL+I
0481      IC2=IC1+IN
0482      STD(IC1)=(STD(IC1)/CLIGHT)*1000.00
0483      STD(IC2)=(STD(IC2)/CLIGHT)*1000.00
0484      WRITE(6,226) STD(IC1),STD(IC2)
0485      226 FORMAT(4X,2F16.12)
0486      76 CONTINUE
      C
0487      STOP
0488      DEBUG SUBCHK
0489      END
```

```
0001      SUBROUTINE MADD(A,B,L,M)
          C
          C
          C      A = A + B
          C      L*M  L*M  L*M
          C
          C
0002      IMPLICIT REAL*8(A-H,O-Z)
0003      DIMENSION A(L,M),B(L,M)
0004      DO 5 I=1,L
0005      DO 5 J=1,M
0006      A(I,J)=A(I,J)+B(I,J)
0007      5  CONTINUE
0008      RETURN
0009      END
```

ORIGINAL PAGE IS
OF POOR QUALITY


```
0001      SUBROUTINE MATPRO(A,B,C,L,M,N)
          C
          C
          C      C = A * B
          C      L*N   L*M M*N
          C
0002      IMPLICIT REAL*8(A-H,O-Z)
0003      DIMENSION A(L,M),B(M,N),C(L,N)
0004      DO 5 I=1,L
0005      DO 5 J=1,N
0006      C(I,J)=0.D0
0007      DO 5 K=1,M
0008      C(I,J)=C(I,J)+A(I,K)*B(K,J)
0009      5      CONTINUE
0010      RETURN
0011      END
```

```

0001      SUBROUTINE TRAPRO(A,B,C,L,M,N)
          C
          C
          C      T
          C      C = A * B
          C      L*N  L*M M*N
          C
          C
0002      IMPLICIT REAL*8(A-H,O-Z)
0003      DIMENSION A(M,L),B(M,N),C(L,N)
0004      DO 5 I=1,L
0005      DO 5 J=1,N
0006      C(I,J)=0.D0
0007      DO 5 K=1,M
0008      C(I,J)=C(I,J)+A(K,I)*B(K,J)
0009      5      CONTINUE
0010      RETURN
0011      END
    
```

ORIGINAL PAGE IS
OF POOR QUALITY

```

0001          SUBROUTINE PROTRA(A,B,C,L,M,N)
          C
          C
          C          T
          C          C = A * B
          C          L*N   L*M M*N
          C
0002          IMPLICIT REAL*8(A-H,O-Z)
0003          DIMENSION A(L,M),B(N,M),C(L,N)
0004          DO 5 I=1,L
0005          DO 5 J=1,N
0006          C(I,J)=0.D0
0007          DO 5 K=1,M
0008          C(I,J)=C(I,J)+A(I,K)*B(K,J)
0009          5      CONTINUE
0010          RETURN
0011          END

```